Atmospheric Radiative Transfer

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Sommaire

1. Introduction

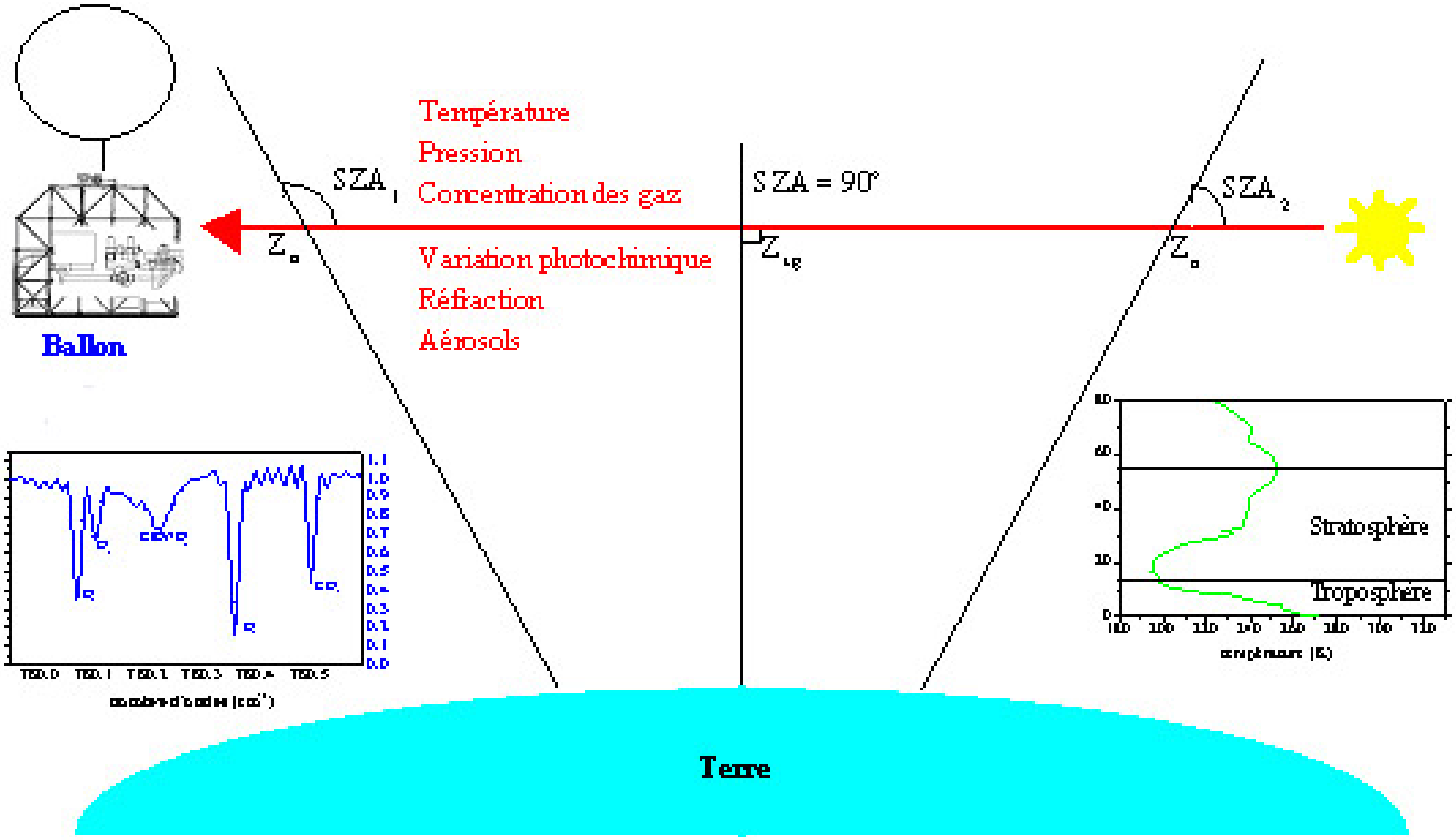
Nomenclature of measurements : Actives vs. passives. In situ vs. remote sensing. ***Remote sensing assignments :*** Inventory (e.g. ground occupancy) Analyze (meteorology, …) Forecast (climate change, …) ***Issues :*** Scientifics Economics Politics Militaries	***Targets :*** Air quality Meteorology Tropospheric chemistry Climate ***Key parameters :*** Temperature and pressure Trace gas concentrations Continental and oceanic surface albedo Vegetation index Clouds (altitude, type, thickness, …) Aerosols (nature, size, amount, …) Solar constants variation with time

Nowadays, remote sensing technics are mainly based on :

- Transmitted or backscattered solar light
- Emitted microwave or thermal radiation (Earth, atmosphere).
- Laser source backscattered (Lidar)

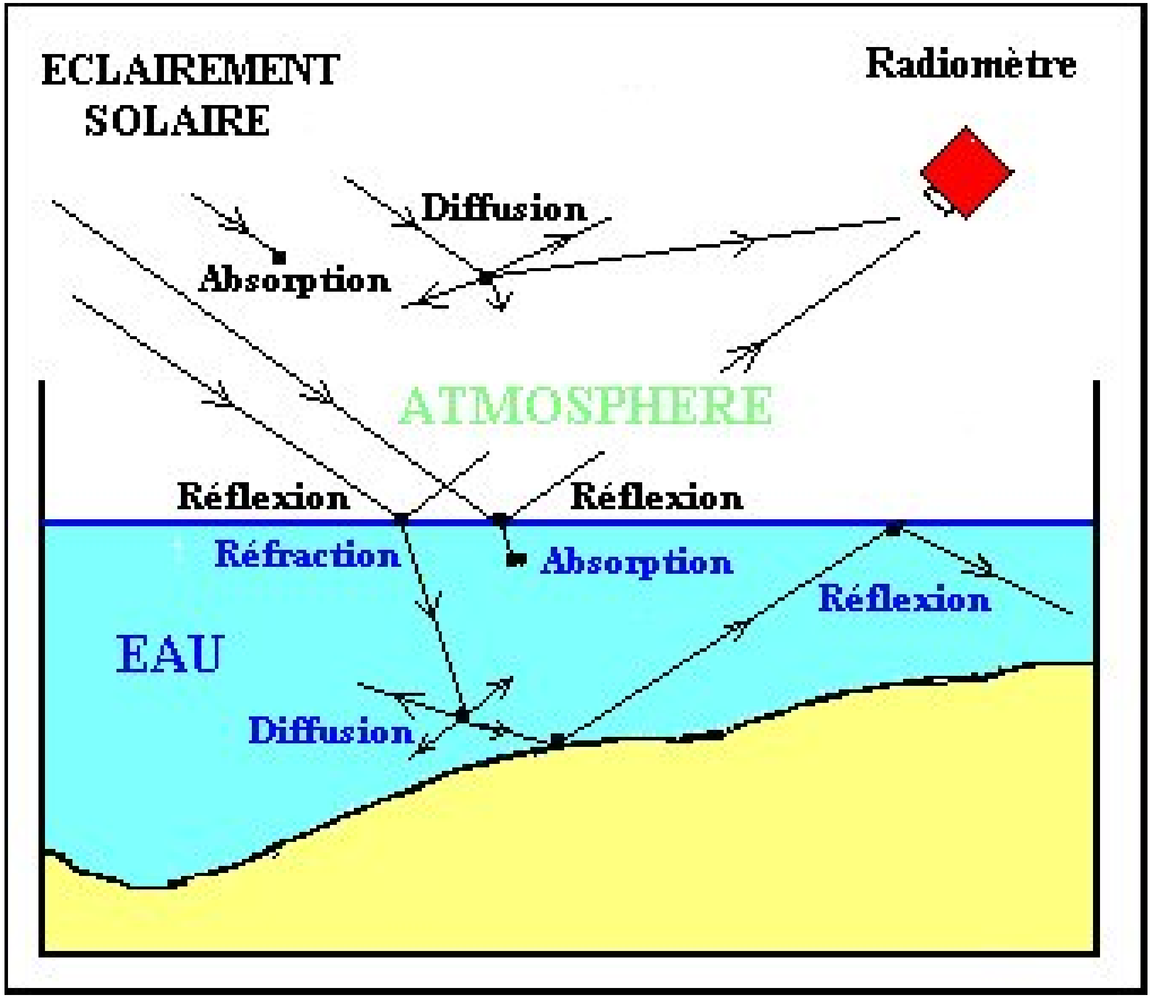
In broad outline : **Light Source** ⇨ **Atmosphere** ⇨ **measurement setup**

⇨ **Need to reproduce accurately radiative transfer processes**



2. Radiation-matter interactions

Any body, having temperature different from Kelvin's zero, is emitting radiation characteristic of its temperature
This radiation can interact with various media he is encountered.
Then, an instrumental setup is receiving an energetic flux which contributions could be various.



[1]

Energy conservation law could be written as :

$$E_{absorption} + E_{réflexion} + E_{transmission} = E_{incident}$$

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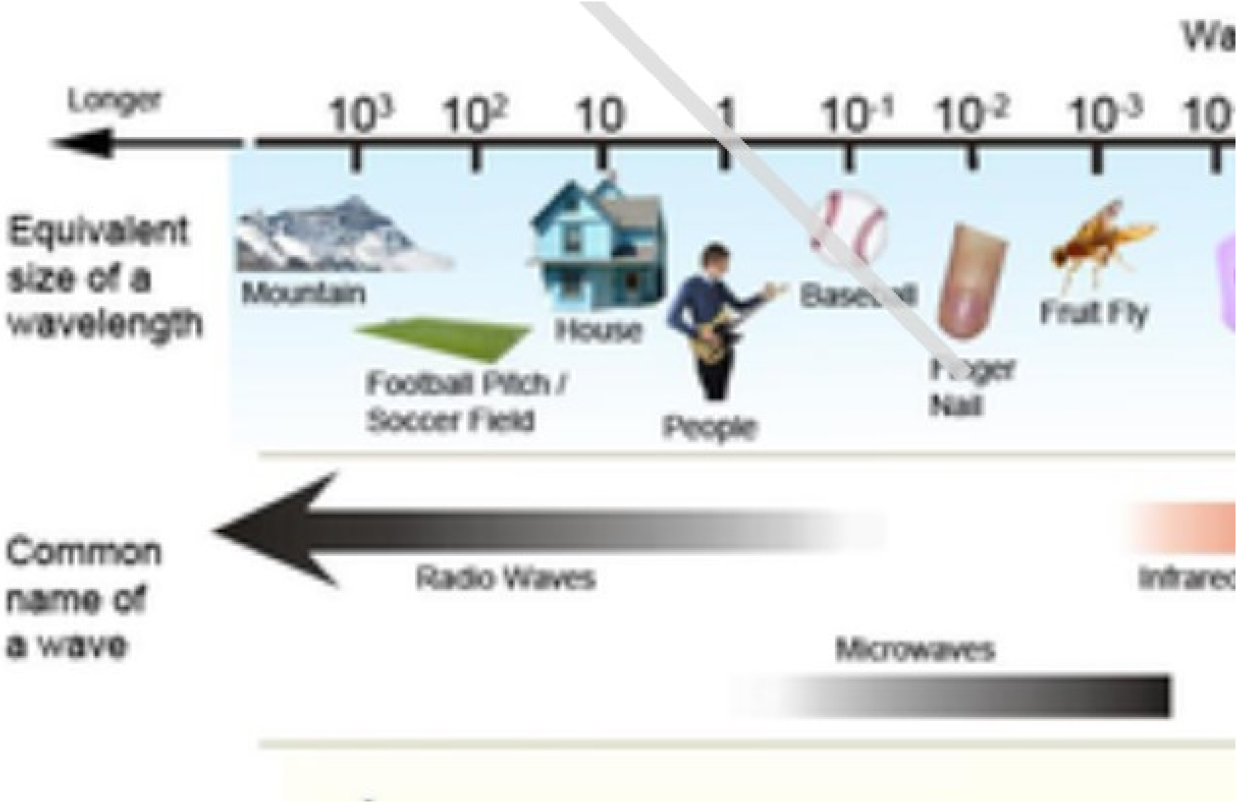
[1] http://www.educnet.education.fr/

2.1 Spectral variables



Relationship between the three spectral parameters :

$$\tilde{\upsilon} = \frac{1}{\lambda} \qquad \upsilon = \frac{c}{\lambda} \qquad \upsilon = c\tilde{\upsilon}$$

We usually expressed λ in microns, $\tilde{\upsilon}$ in cm^{-1}, and ν in Hz.
Convenient :relationship :

$$\tilde{\upsilon}\left[cm^{-1}\right] = \frac{10^4}{\lambda[\mu m]} \qquad \upsilon\,[Hz] = \frac{2{,}998\times10^{14}}{\lambda[\mu m]}$$

Please note that we use in this course alternatively wavelength and wave number.

2.2 Energetic flux

Amount of energy radiated by a source during a time unit in the whole space (so it is a power)

$$\Phi = \frac{dE\ (t)}{dt}$$

Spectral energetic flux

$$\Phi_\lambda = \frac{dE_\lambda(t)}{dt}$$

2.3 Radiant emittance of a source / irradiance of a surface

Several terms could be used for radiometric amount. Then, we can talk of *radiant exitance* or *radiant emittance* of a source, and of *irradiance* received by an object (respectively *emittance* and *eclairement* in French).

It is the energetic flux radiated per unit area in all directions of space outside the source.

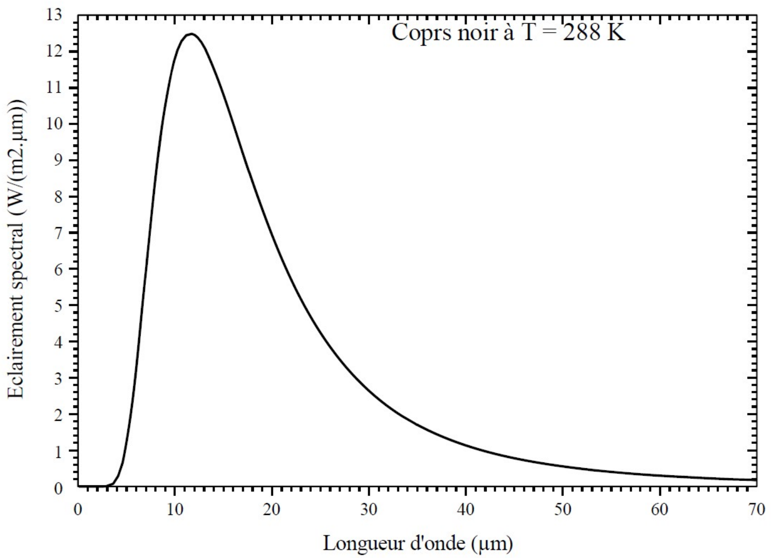
$$M = \frac{d\Phi_{ext}}{dS}$$

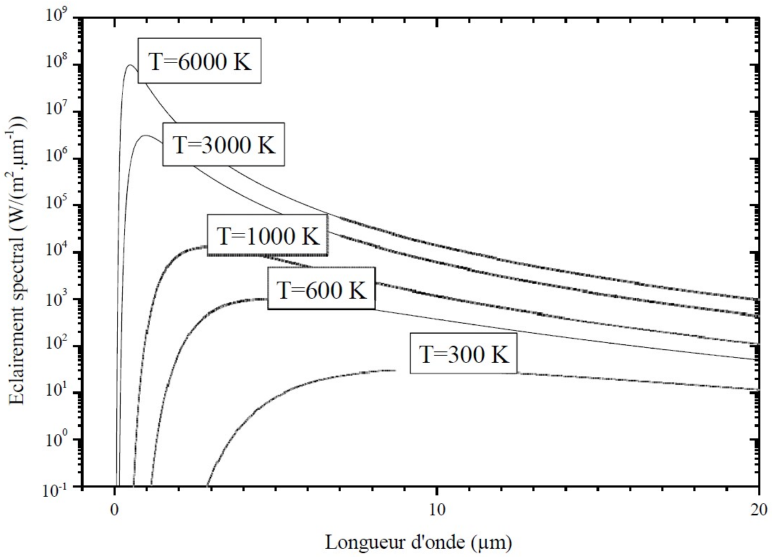
If the source is a sphere of radius R, and if the radiation is isotropic, the emittance was then expressed as:

$$M = \frac{\Phi_{ext}}{4\pi R^2}$$

Spectral emittance :

$$M_\lambda = M(\lambda) = \frac{dM}{d\lambda} \qquad M = \int_0^{+\infty} M(\lambda)\,d\lambda$$





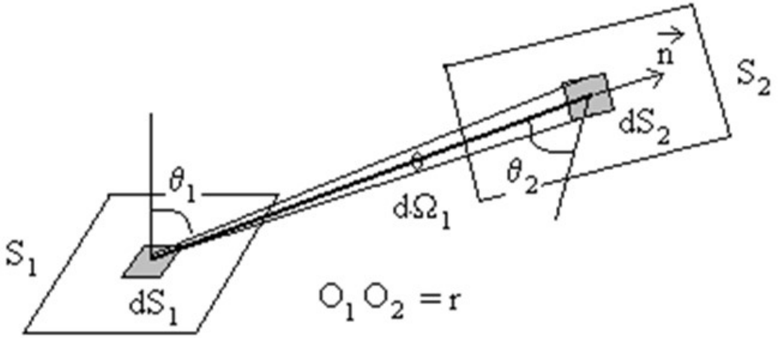
2.4 Lambert law.

A source responds Lambert's law if the energy radiated from a point of this source is the same in all directions (it is said that its intensity is isotropic and therefore independent of the angle from which we observe this source).

Defining M as the value of the illumination measured by a sensor, one can easily deduce the energy flux of the source shaving a surface S : $\Phi = S\,M$

2.5 Radiance (Luminance énergétique)

Consider a non-point source to which energy emittance M is known at any point. It is possible that intensity emitted from a surface element dS of the source is not isotropic and therefore varies with the observation viewing angle θ to the collinear direction indicated by the vector $d\vec{S}$.



Let dS_1 and dS_2 be elementary surfaces of the surfaces S_1 and S_2 of two separate radiating body apart of distance $r = O_1O_2$. The radiant energetic flux $d\Phi$ emitted from the surface dS_1 towards the surface dS_2 (direction $\vec{n}$) is equal to :

$$d^2\Phi = L(\theta) dS_1 \cos\theta_1 \, d\Omega_1$$

By definition, L is the radiance of the surface, and $d\Omega_1 = \frac{dS_2 \cos\theta_2}{r^2}$ is the solid angle under which is seen dS_2 from point O_1.

More generally, energy radiated by a body in a direction, forming an angle θ with the normal to the emission surface, is lower than the one radiated in the direction of the vector $d\vec{S}$.
So by simple application of trigonometry we get the definition of radiance (or *luminance* in French) :

$$L(\theta) = \frac{dM}{d\Omega \cos(\theta)} = \frac{d^2\Phi}{dS \, d\Omega \cos(\theta)}$$

We can also write :

$$M = \int_{\Sigma} L(\theta) \cos(\theta) d\Omega$$

This gives us the energetic intensity that radiates a source of radiance L in a hemisphere.

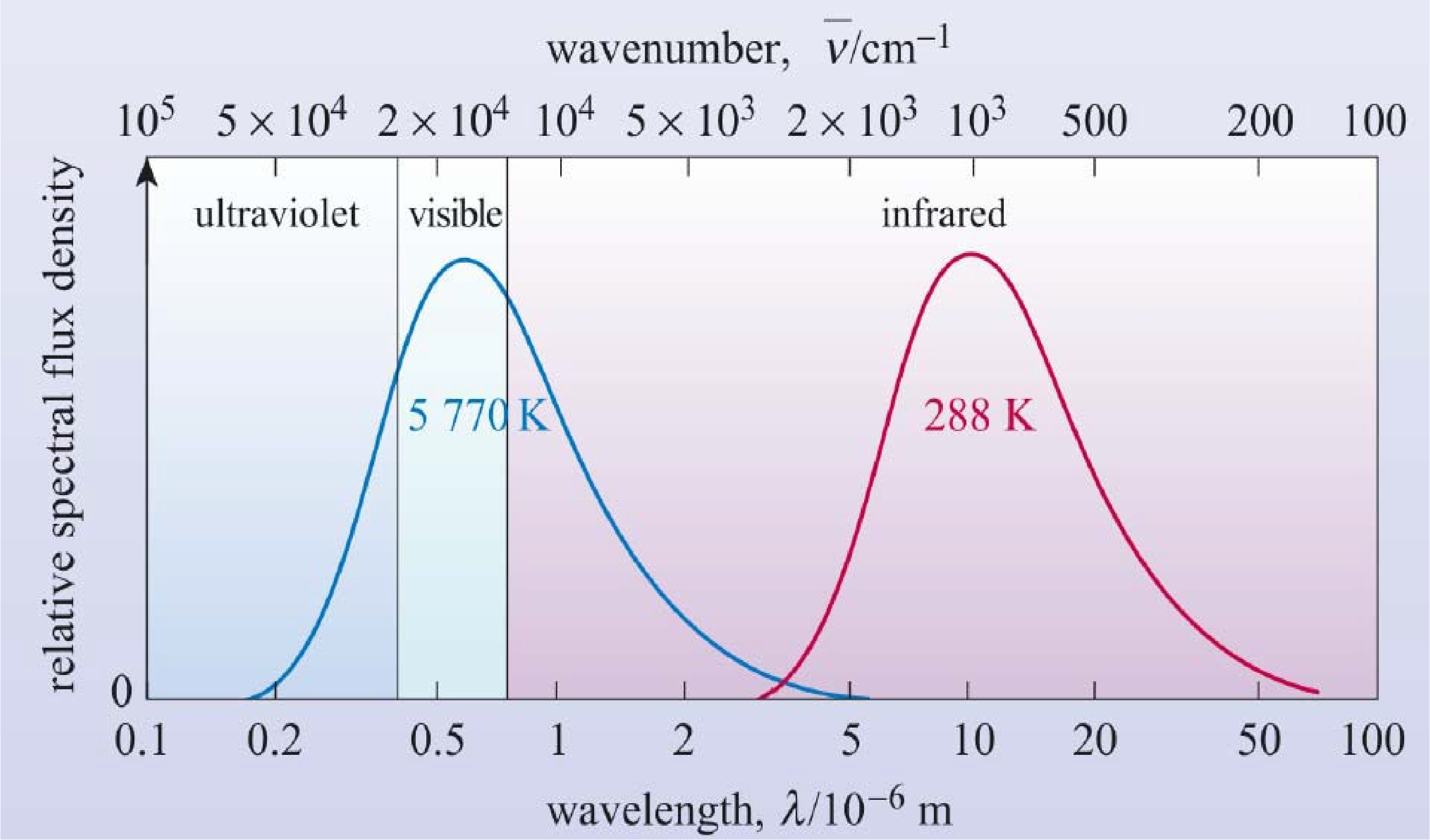
Remarque : la luminance d'une source est quelque fois appelée "brillance" ou "éclat" lorsqu'on ne considère que le rayonnement visible. Elle est également parfois appelée intensité spécifique, en anglais *radiance*.

There is the special case of Lambertian body for which the emittance is :

$$M = \frac{\Phi}{S} = \frac{I_0 \pi}{S} = L\pi$$

This result is significant because the value of the radiance measured by a sensor allow to infer emittance M and therefore energetic flux from the source :

$$\Phi = S\,L\pi$$



2.6 Optical depth

Consider a radiation of spectral radiance $L_\lambda(0)$ propagating rectilinearly along a curvilinear *Os* axis arriving normally on a layer of material of thickness D. Part of this radiation is absorbed during its path.

Electromagnetic theory predicts that this decrease in radiance is proportional to $L_\lambda(0)$:

$$dL_\lambda(s) = -\alpha_\lambda L_\lambda(s)\, ds \qquad \text{or} \qquad \frac{dL_\lambda(s)}{L_\lambda(s)} = -\alpha_\lambda\, ds$$

where α_λ is the absorption coefficient of the medium (dimension of the reciprocal of length)

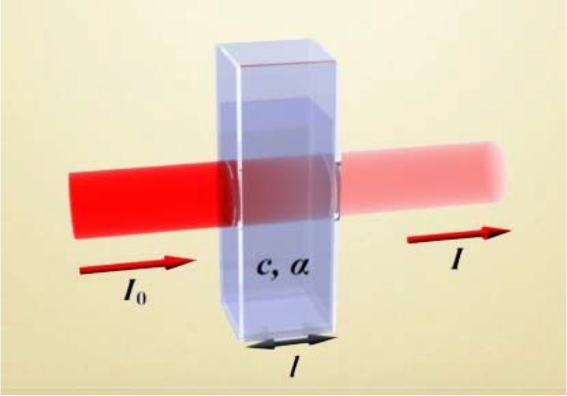
If α_λ is independent of s, we can write :

$$\int_{L_\lambda(0)}^{L_\lambda(D)} \frac{dL_\lambda(s)}{L_\lambda(s)} = -\alpha_\lambda \int_0^D ds \Rightarrow \ln\left(\frac{L_\lambda(D)}{L_\lambda(0)}\right) = -\alpha_\lambda D \Rightarrow L_\lambda(D) = L_\lambda(0)\, e^{-\alpha_\lambda D}$$

An exponential decay is obtained: this is the law of Bouguer, Lambert Beer.

If α_λ depend of s, we obtain a more complex expression that we do not consider later :

$$\int_{L_\lambda(0)}^{L_\lambda(D)} \frac{dL_\lambda(s)}{L_\lambda(s)} = -\int_0^D \alpha_\lambda ds \Rightarrow L_\lambda(D) = L_\lambda(0)\, e^{-\int_0^D \alpha_\lambda ds}$$



The Beer-Lambert absorption (in the absence of scattering and own emission of medium) can also be expressed for Radiance (homogeneous medium):

$$L_\lambda(D) = L_\lambda(0) \exp\left(-\tau_\lambda(D,0)\right)$$

where $\tau_\lambda(D,0)$ is the optical thickness between the point of abscissa 0 and that of abscissa D.

For pure gas at the temperature T and pressure P absorbing in the vicinity of wavelength λ :

$$\tau_\lambda(D,0) = N\, D\, \sigma(\lambda, P, T)$$

N = number density	molecule.cm^{-3}
D = path length	cm
$\sigma(\lambda, P, T)$ = absorption cross section	cm^{-2}/molecule

Recall that for an ideal gas : $N = L \dfrac{P}{P_0} \dfrac{T_0}{T}$

P_0 = 1013.25 hPa = 1 atm

T_0 = 273.15 K

$L = 2.6867775 \times 10^{19}$ molecule.cm^{-3}

2.7 Line spectra (line by line calculation)

When we take into account a line spectrum (almost always in the infrared and microwave) expression of the optical thickness is written :

$$\tau_{\tilde{\nu}}(s) = \sum_i \sum_j \int_0^s k_{\tilde{\nu}_j}^{N_i}(T(s))\Phi\left(\tilde{\nu} - \tilde{\nu}_j, P(s), T(s)\right)N_i(s)\,ds$$

In this equation :

- i is an index that characterizes the various absorbing species.
- j is the index that characterizes each line (transition) of species i
- $N_i(s)$ is the number density (in molecule/cm^3) of species i.

But instead of using cross sections σ_i valid for a continuous spectrum, one must consider the intensity $k_{\tilde{\upsilon}_j}^{N_i}$ of the line j centered at position $\tilde{\upsilon}_j$ of species i.

In the spectroscopic databases $k_{\tilde{\upsilon}_j}^{N_i}$ is given as standard in cm^{-1}/(molecule cm^{-2}) at reference temperature T_{ref} = 296K. A At local thermodynamic equilibrium, knowing the lower energy level of the transition (E" en cm^{-1}), one can convert this intensity to the desired temperature by the formula:

$$k_{\tilde{\nu}_j}^{N_i}(T) = k_{\tilde{\nu}_j}^{N_i}(T_{ref})\,\frac{Z(T_{ref})}{Z(T)}\,\exp\left[\frac{-h\,c\,E''}{k}\left(\frac{1}{T} - \frac{1}{T_{ref}}\right)\right]\frac{1-\exp\left(\dfrac{-h\,c\,\tilde{\nu}}{k\,T}\right)}{1-\exp\left(\dfrac{-h\,c\,\tilde{\nu}}{k\,T_{ref}}\right)}$$

where :

- Z(T) is the total partition function and where we took into account the temperature dependence of the stimulated emission term.
- $\Phi(\Delta\tilde{\upsilon}, \mathrm{P}, \mathrm{T})$ is the line shape of the corresponding line (spectral profile), which depends on both pressure and temperature.

The line shape is normalized so that : $\int_{-\infty}^{+\infty} \Phi(\Delta\tilde{\nu}, \mathrm{P}, \mathrm{T})\, d\Delta\tilde{\nu} = 1$

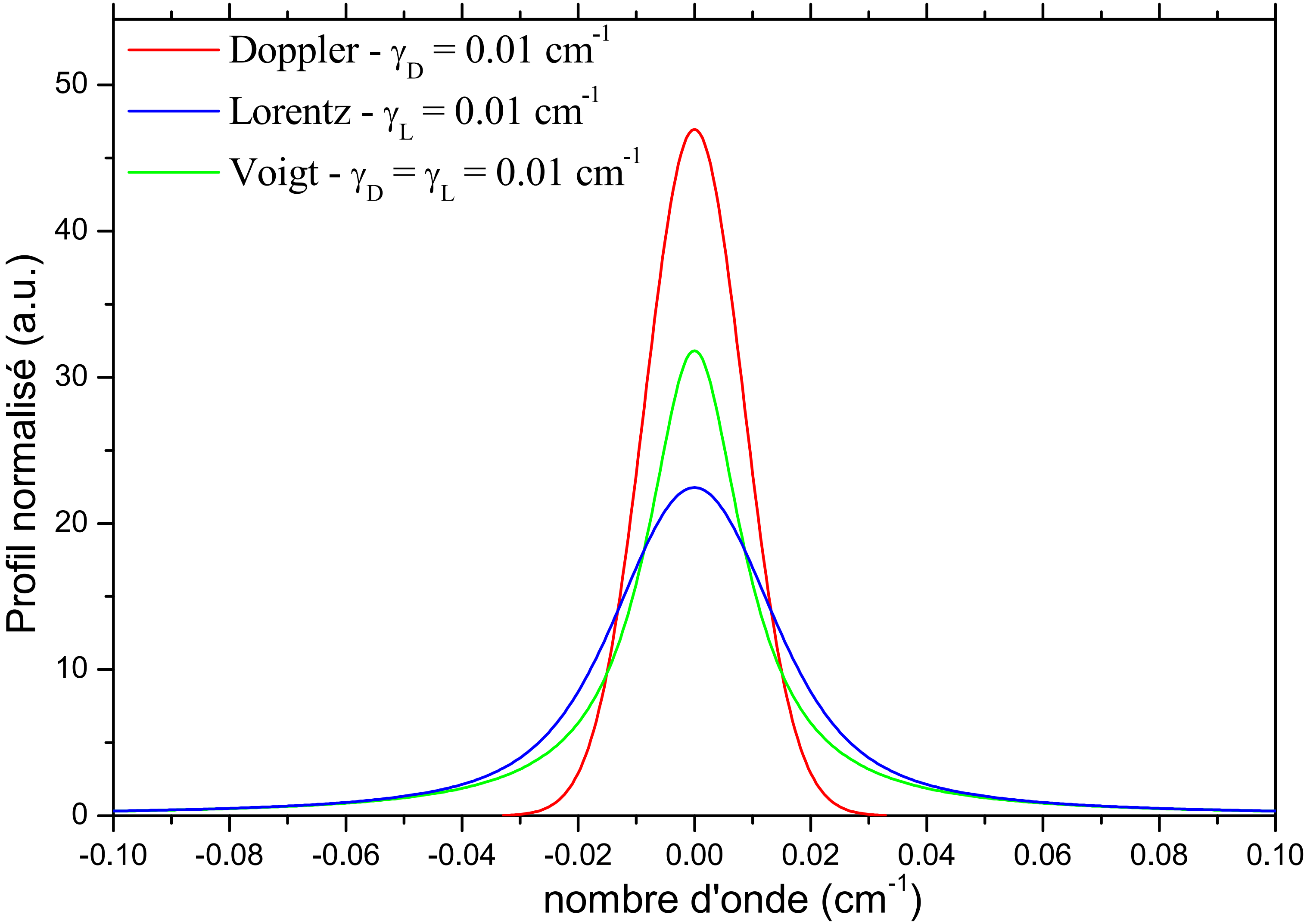
Spectroscopy courses provide information on molecular spectra and the intensities and line profiles that must be considered in the Earth's atmosphere.

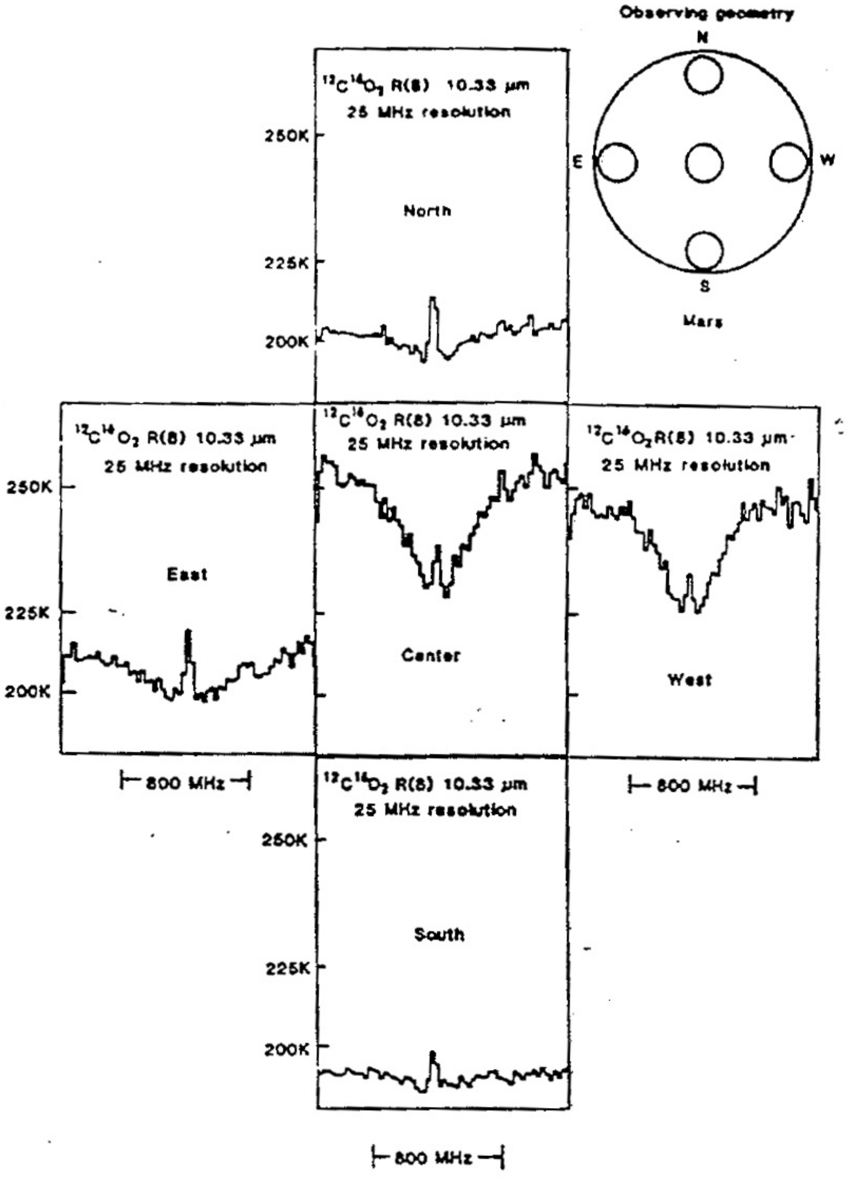
Calculation of optical thickness must consider both the rapid dependence of $\tau_{\tilde{\upsilon}}$ as a function of $\tilde{\upsilon}$ (hence the term of line by line calculation). One must, for the integration along the line of sight, proceeding to a **division of the atmosphere into layers** sufficiently thin so that **rapid changes in intensity** $k_{\tilde{\upsilon}}^{N}(\mathrm{T}(s))$ **and line shapes** $\Phi(\Delta\tilde{\upsilon}, \mathrm{P}(s), \mathrm{T}(s))$ **are well taken into account** in the integration on the space variable s.

Although this expression of the transmission is complicated, we understand that, since the concentrations **N$_i$(s) occurred**, one could in principle by a mathematical process described under the general term ***inverse method*** **infer concentration profiles N$_i$(z) from transmissions** $\mathrm{T}_{\tilde{\upsilon}}(s,0)$.

For example if one has spectra of atmospheric absorption, and then $\mathrm{T}_{\tilde{\upsilon}}(\mathrm{H}_t)$ for several tangent altitudes in a solar occultation sequence (cf. § 4.2) one can expect infer concentration vertical profiles of different species absorbing in a given spectral region. An example in the form of exercise is given at the end of the course.

Importance of the line shape :





CO$_2$ line profile map of Mars Feb. 1980

CO$_2$ remote sensing by IR heterodyne spectroscopy at 10 μm in Mars atmosphere (Mumma et al., Science 212, 1981).

We distinguish clearly:
- surface with a variable continuum depending on the geographical position
- troposphere with an enlarged collision line
- mesosphere with a very narrow central contribution.

2.8 continuous spectra

In the visible and UV we will often consider continuous spectra (electronic transitions toward dissiociatif states, or dense spectra) and we write :

$$\tau_{\tilde{\nu}}(s) = \sum_i \int_0^s \sigma_i(\tilde{\nu}, P(s), T(s)) N_i(s) ds$$

In this equation :

- i is an index that characterizes the various absorbing species.
- $N_i(s)$ is the number density (en molecule/cm^3) of species i.
- $\sigma_i[\tilde{\upsilon}, T(s), P(s)]$ is the absorption cross section in cm^2/molecule for species i at wavenumber $\tilde{\upsilon}$.

We make explicit the possibility of a dependence of the absorption cross section as a function of pressure and/or the local temperature.
As a first approximation (for ozone in the UV for example) cross sections do not depend on P and we can write that $\sigma_i = \sigma_i[\tilde{\upsilon}, T]$. In addition, if dependencies on the temperature are negligible (which is not necessarily the case of ozone in the UV) you can even write $\sigma_i[\tilde{\upsilon}]$.
Under these conditions the measurement of the transmission provides direct access to the quantities integrated along the line of sight :

$$\tau_{\tilde{\upsilon}}(s,0) = \sum_i \sigma_i[\tilde{\upsilon}] Q_i$$

where the integrated amount for the species i expressed in molecule/cm^2 is written :

$$Q_i = \int_0^s N_i(s)ds$$

We understand that making measurements in suitable spectral bands (multiple values of $\tilde{\upsilon}_k$) associated with different values of $\sigma_i[\tilde{\upsilon}_k]$ we can have a sufficient number of equations of the type :

$$\tau_{\tilde{\upsilon}_k}(s,0) = \sum_i \sigma_i[\tilde{\upsilon}_k]Q_i$$

Allowing to determinate separately the Q_i .

Remarks :

- Continua
- Collision induced spectra

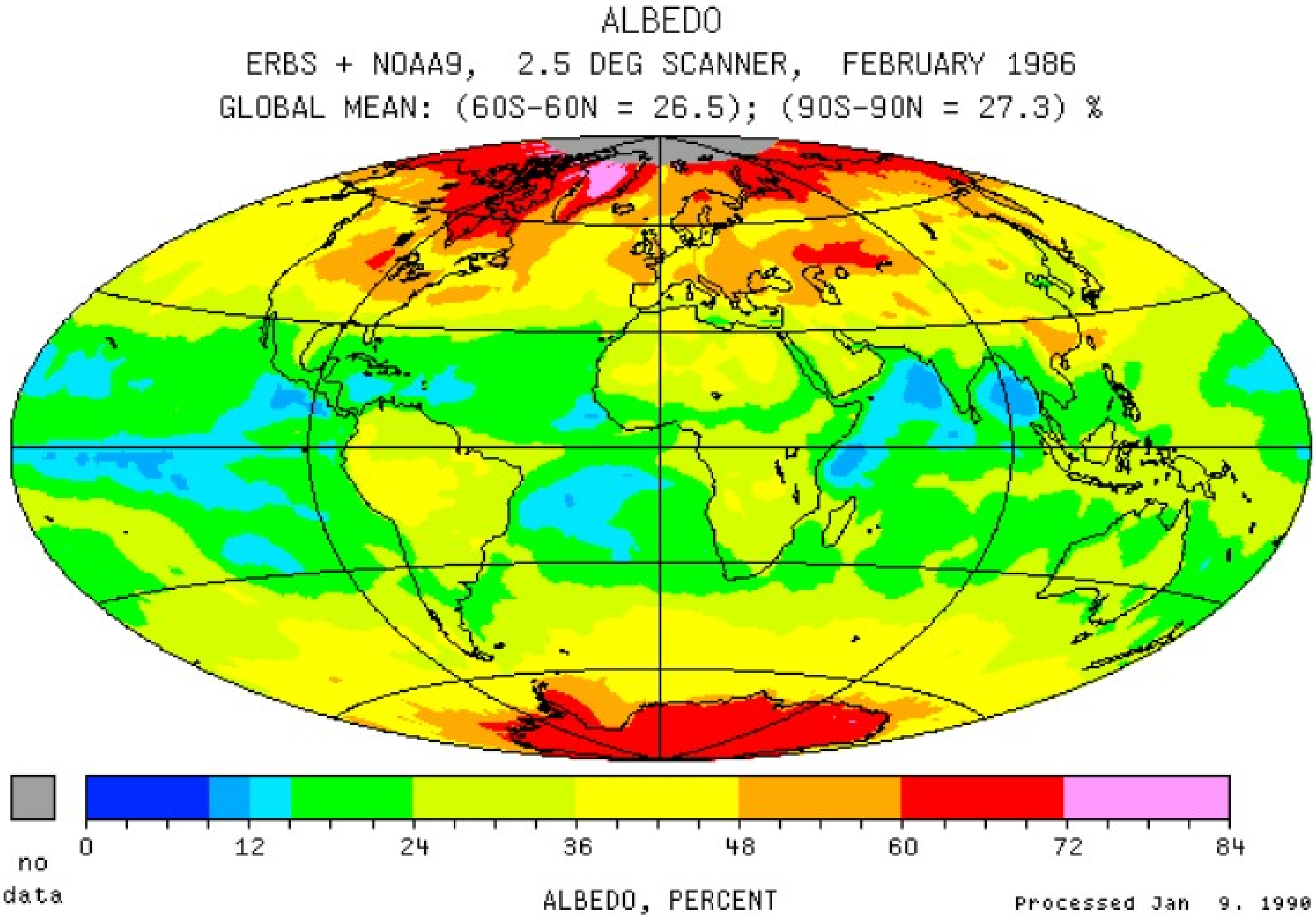
2.9 Albedo

Called albedo the ratio of reflected energy E_r and the incident energy E_i : $\gamma = E_r/E_i$ or $\gamma = {}^{\Phi_r}/_{\Phi_i}$

The albedo depends mainly on the angle of incidence of radiation and the nature of the receiving surface. The albedo of a surface can vary significantly with the wavelength of the radiation. Earth surface albedo varies greatly from one point to another in the world.

Type de surface	**Albedo (0 à 1)**
Mer calme (soleil au zenith)	0,10
Sommet de certains nuages	0,70
Forêt équatorial	0,10
Surface de lac	0,02 à 0,04
Forêt de conifères	0,05 à 0,15
Surface de la mer	0,05 à 0,15
Sol sombre	0,05 à 0,15
Cultures	0,15 à 0,25
Sable léger et sec	0,25 à 0,45
Glace	0,30 à 0,40
Neige tassée	0,40 à 0,70
Neige fraîche	0,75 à 0,90

In practice, a body is seen as white when it reflects at least 80% of light of a white light source. In contrast while reflecting less than 3% of the incident light body appears black.


ALBEDO
ERBS + NOAA9, 2.5 DEG SCANNER, FEBRUARY 1986
GLOBAL MEAN: (60S-60N = 26.5); (90S-90N = 27.3) %
no data
0
12
24
36
48
60
72
84
ALBEDO, PERCENT
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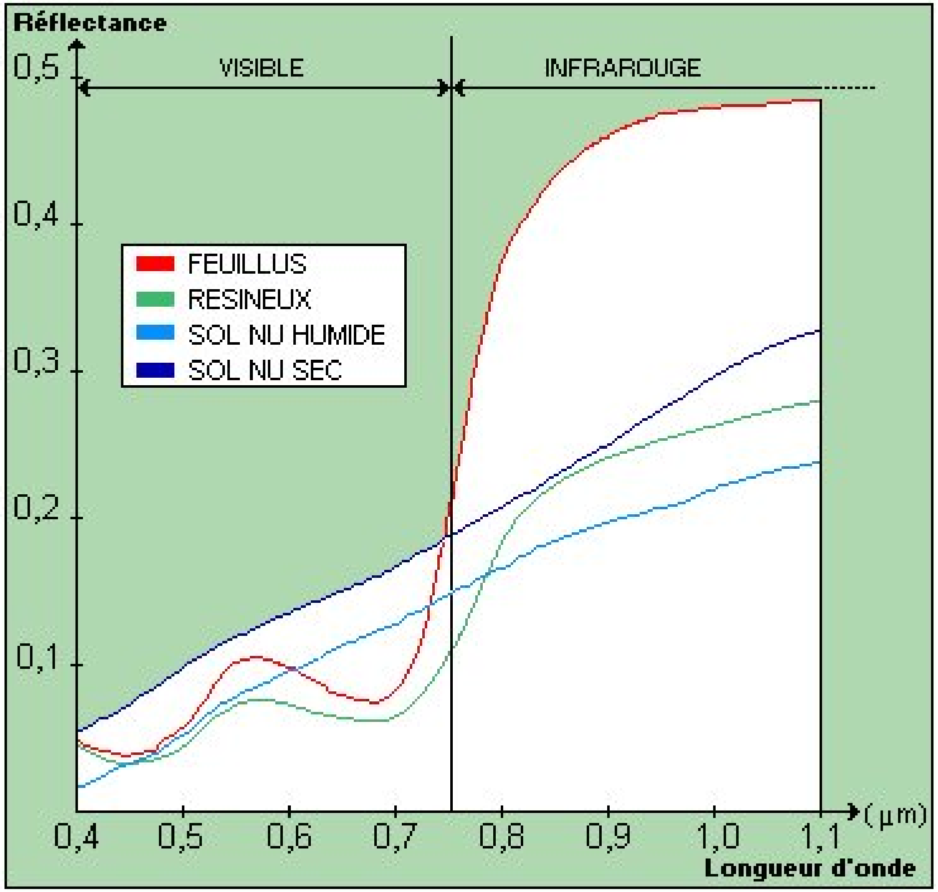
2.10 Reflectance

Reflectance or reflexivity = (reflected energy)/(incident energy)



Reflectance as a function of the medium.

(Feuillu = leafy Résineux = resiniferous Sol nu humide = wet soil Sol nu sec = dry soil)

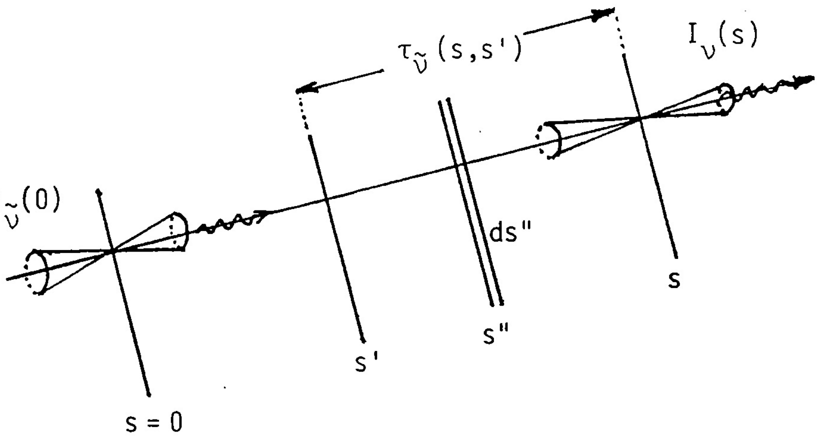
3. Radiative transfer equation

3.1 Radiative transfer equation neglecting scattering

If we take the previous expression of the radiative transfer equation which assumes negligible scattering (which is true in the infrared), we obtain :

$$\frac{d L_\lambda(s)}{d s} = -\alpha_\lambda(s)\left(L_\lambda(s) - J_\lambda(T(s))\right)$$

This is the local form of the equation of radiative transfer when scattering is neglected.



The integrated form of the radiative transfer equation can also be written after integration between points 0 and s:

$$L_\lambda(s) = L_\lambda(0)\exp\left[-\tau_\lambda(s,0)\right] + \int_0^S J_\lambda(s')\exp\left[-\tau_\lambda(s,s')\right]\alpha_\lambda(s')ds'$$

Where optical depth $\tau_{\tilde{\upsilon}}(\mathrm{s,s'})$ between the coordinate points along s and s' of the path followed by the radiation i.e. line of sight, LOS) is defined by :

$$\tau_\lambda(\mathrm{s,s'}) = \int_{\mathrm{s'}}^{\mathrm{s}} \alpha_\lambda(\mathrm{s''})\mathrm{ds''}$$

You can check by differentiating that the integrated form is solution of the local form of the equation of radiative transfer.

The above equation has two terms:

The first term expresses the propagation in the medium of the incident radiation (to the point s = 0). As a result of the absorption processes the beam is attenuated by a factor:

$$\mathrm{t}_\lambda(\mathrm{s},0) = \exp\left[-\tau_\lambda(\mathrm{s},0)\right]$$

which can also be called transmittance or transmission of medium from 0 to s.

The second term reflects the contribution to the intensity at the point s of all elementary layers between 0 and s. The Emission of a layer of thickness *ds'* is attenuated by a factor :

$$\mathrm{t}_\lambda(\mathrm{s,s'}) = \exp\left[-\tau_\lambda(\mathrm{s,s'})\right]$$

resulting from absorption between S 'and S of the intensity emitted at the point S'.

From the general expression for the transmission

$$\mathrm{t}_\lambda(\mathrm{s,s'}) = \exp\left[-\int_{\mathrm{s'}}^{\mathrm{s}} \alpha_\lambda(\mathrm{s''})\mathrm{ds''}\right]$$

We deduce

$$\frac{\partial t_{\lambda}(s,s')}{\partial s} = -\alpha_{\lambda}(s)t_{\lambda}(s,s') \qquad \frac{\partial t_{\lambda}(s,s')}{\partial s'} = -\alpha_{\lambda}(s')t_{\lambda}(s,s')$$

So that the integrated form of the transfer equation can be written as:

$$L_{\lambda}(s) = L_{\lambda}(0)\, t_{\lambda}(s,0) + \int_0^S J_{\lambda}(s')\, \frac{\partial t_{\lambda}(s,s')}{\partial s'} ds'$$

According to the spectral region and in the geometry used it is possible that only the first or only the second of the two terms of the equation is dominant. In the first case we speak of absorption, in the second issue of emission. But the equation has a general validity to cover cases where the emission and absorption phenomena must be considered simultaneously.

3.2 Absorption

If we consider only the absorption (thus neglecting emission and scattering), if medium is illuminated by radiance $L_{\tilde{\nu}}(0)$ at entrance of the line of sight, output radiance is then:

$$L_{\tilde{\nu}}(s) = L_{\tilde{\nu}}(0)\; t_{\tilde{\nu}}(s,0)$$

With :

$$t_{\tilde{\nu}}(s,0) = \exp[-\tau_{\tilde{\nu}}(s,0)]$$

Specific cases :

$\tau_{\tilde{\nu}}(s,0) \ll 1$ medium optically thin $L_{\tilde{\nu}}(s) = L_{\tilde{\nu}}(0)[1 - \tau_{\tilde{\nu}}(s,0)]$

$\tau_{\tilde{\nu}}(s,0) \gg 1$ medium optically thick $L_{\tilde{\nu}}(s) = 0$

3.3 Emission

At local thermodynamic equilibrium (LTE), the source term $J_{\tilde{\nu}}$ is equal to the Planck function $B_{\tilde{\nu}}$:

$$L_{\tilde{\nu}}(s) = \int_0^s B_{\tilde{\nu}}(T(s')) \frac{\partial t_{\tilde{\nu}}(s,s')}{\partial s'} ds'$$

For homogeneous and isotherm medium :

$$L_{\tilde{\nu}}(s) = B_{\tilde{\nu}}(T)[1 - t_{\tilde{\nu}}(s,0)]$$

with

$$t_{\tilde{\nu}}(s,0) = \exp[-\tau_{\tilde{\nu}}(s,0)]$$

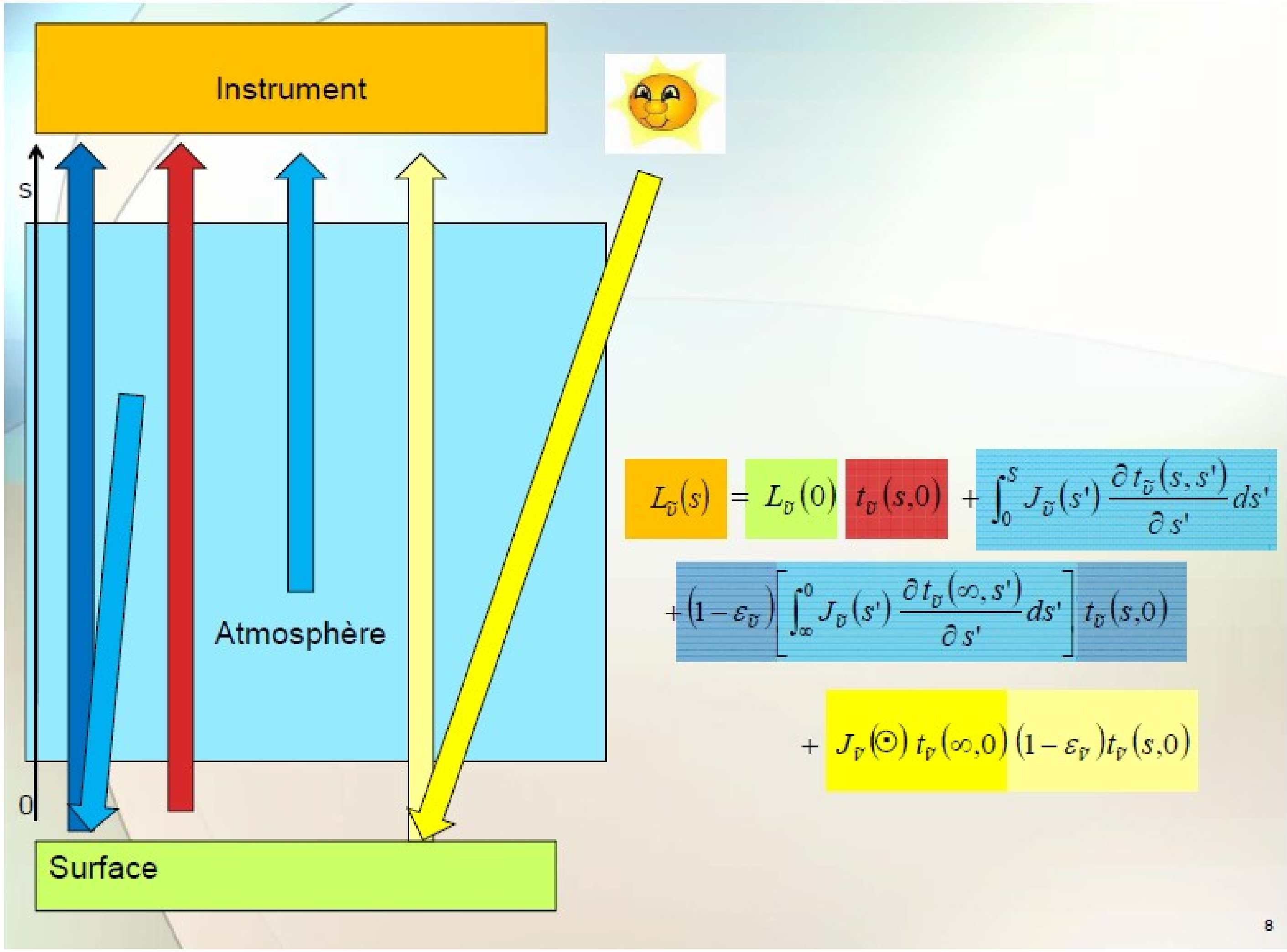
and

$$t_{\tilde{\nu}}(s,0) = \exp\left[-\int_0^s \alpha_{\tilde{\nu}}(s'')ds''\right] = \exp[-\alpha_{\tilde{\nu}}\, s]$$

Specific cases :

$\tau_{\tilde{\nu}}(s,0) << 1$ medium optically thin $L_{\tilde{\nu}}(s) = B_{\tilde{\nu}}(\tilde{\nu},T)\tau_{\tilde{\nu}}(s,0)$

$\tau_{\tilde{\nu}}(s,0) >> 1$ medium optically thick $L_{\tilde{\nu}}(s) = B_{\tilde{\nu}}(\tilde{\nu},T)$ (blackbody)



$$L_{\tilde{\upsilon}}(s) = L_{\tilde{\upsilon}}(0)\, t_{\tilde{\upsilon}}(s,0) + \int_0^s J_{\tilde{\upsilon}}(s') \frac{\partial t_{\tilde{\upsilon}}(s,s')}{\partial s'} ds'$$

$$+ (1-\varepsilon_{\tilde{\upsilon}}) \left[\int_\infty^0 J_{\tilde{\upsilon}}(s') \frac{\partial t_{\tilde{\upsilon}}(\infty,s')}{\partial s'} ds' \right] t_{\tilde{\upsilon}}(s,0)$$

$$+ J_{\tilde{\nu}}(\odot)\, t_{\tilde{\nu}}(\infty,0)\, (1-\varepsilon_{\tilde{\nu}}) t_{\tilde{\nu}}(s,0)$$

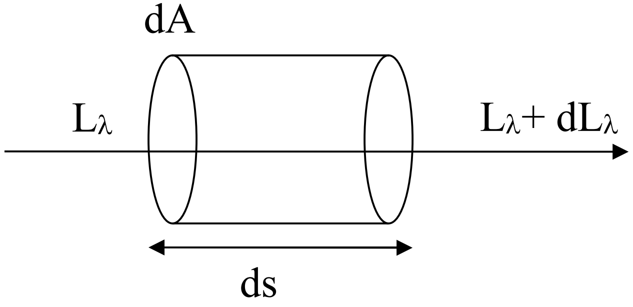
3.4 General equation including scattering

3.4.1 General form

We are looking for an equation describing the spatial distribution (angular) of the radiance at the output of a medium that can be inhomogeneous. We can establish it making an energy assessment.

Be a medium that absorbs, emits, and backscatter radiation, characterized by its spectral absorption coefficient α_λ and by its backscatter spectral coefficient σ_λ. A monochromatic radiation with radiance $L_\lambda(s)$ goes through this medium at time t and in the direction Ω defined by Os curvilinear axis.

Let be dA the elementary section and be ds the length of the elementary cylindrical volume :



Be $L_\lambda(s)$ radiance at point s, and be $L_\lambda(s) + dL_\lambda$ radiance at point s +ds.

$dL_\lambda dA\, d\Omega\, d\lambda\, dt$ is the difference between energies passing through surfaces dA at point s+ds during the time dt in the spectral range dλ around λ, and supported in elementary solid angle dΩ in the direction Ω.

Be W_λ the radiative energy gain around a given direction, given per solid angle unit, per volume unit, and per wavelength unit.

As dV=dAds, we obtain :

$$W_\lambda dA\ ds\ d\Omega\ d\lambda\ dt = dL_\lambda\ dA\ d\Omega\ d\lambda\ dt$$

Resulting that :

$$\frac{d\,L_\lambda(s)}{ds} = W_\lambda(s)$$

W_λ is then the sum of 4 contributions:

$$W_\lambda = W_{\text{émis}} - W_{\text{abs}} - W_{\text{diff.out}} + W_{\text{diff.in}}$$

$W_{\text{émis}}$ is radiative energy gains by the beam thanks to emission in the volume element :

$$W_{\text{emis}} = \alpha_\lambda(s)\,L_\lambda^0(T(s))$$

W_{abs} is the radiative energy losses by the beam due to absorption :

$$W_{\text{abs}} = \alpha_\lambda(s)\,L_\lambda(s)$$

$W_{\text{diff.in}}$ is radiative energy gains by the beam collected in the Ω direction thanks to backscattered radiation coming from all directions :

$$W_{\text{diff-in}} = \frac{1}{4\pi}\sigma_\lambda \int_{4\pi} P_\lambda\left(\vec{\Omega}' \to \vec{\Omega}\right) L_\lambda(s)\,d\Omega'$$

$W_{diff.out}$ is the radiative energy losses by the beam due to scattering of medium in volume dV :

$$W_{diff-out} = \sigma_\lambda L_\lambda(s)$$

Than we can write the radiative transfer equation as :

$$\frac{dL_\lambda(s)}{ds} = \alpha_\lambda(s) L_\lambda^0(T(s)) - \alpha_\lambda(s) L_\lambda(s) - \sigma_\lambda L_\lambda(s) + \frac{1}{4\pi}\sigma_\lambda \int_{4\pi} P_\lambda(\vec{\Omega}' \to \vec{\Omega}) L_\lambda(s) d\Omega'$$

$$\frac{dL_\lambda(s)}{ds} + (\alpha_\lambda(s) + \sigma_\lambda) L_\lambda(s) = \alpha_\lambda(s) L_\lambda^0(T(s)) + \frac{1}{4\pi}\sigma_\lambda \int_{4\pi} P_\lambda(\vec{\Omega}' \to \vec{\Omega}) L_\lambda(s) d\Omega'$$

Introducing monochromatic volumic extinction coefficient : $\beta_\lambda = \alpha_\lambda + \sigma_\lambda$

And monochromatic albedo $\omega_\lambda = \frac{\sigma_\lambda}{\beta_\lambda}$

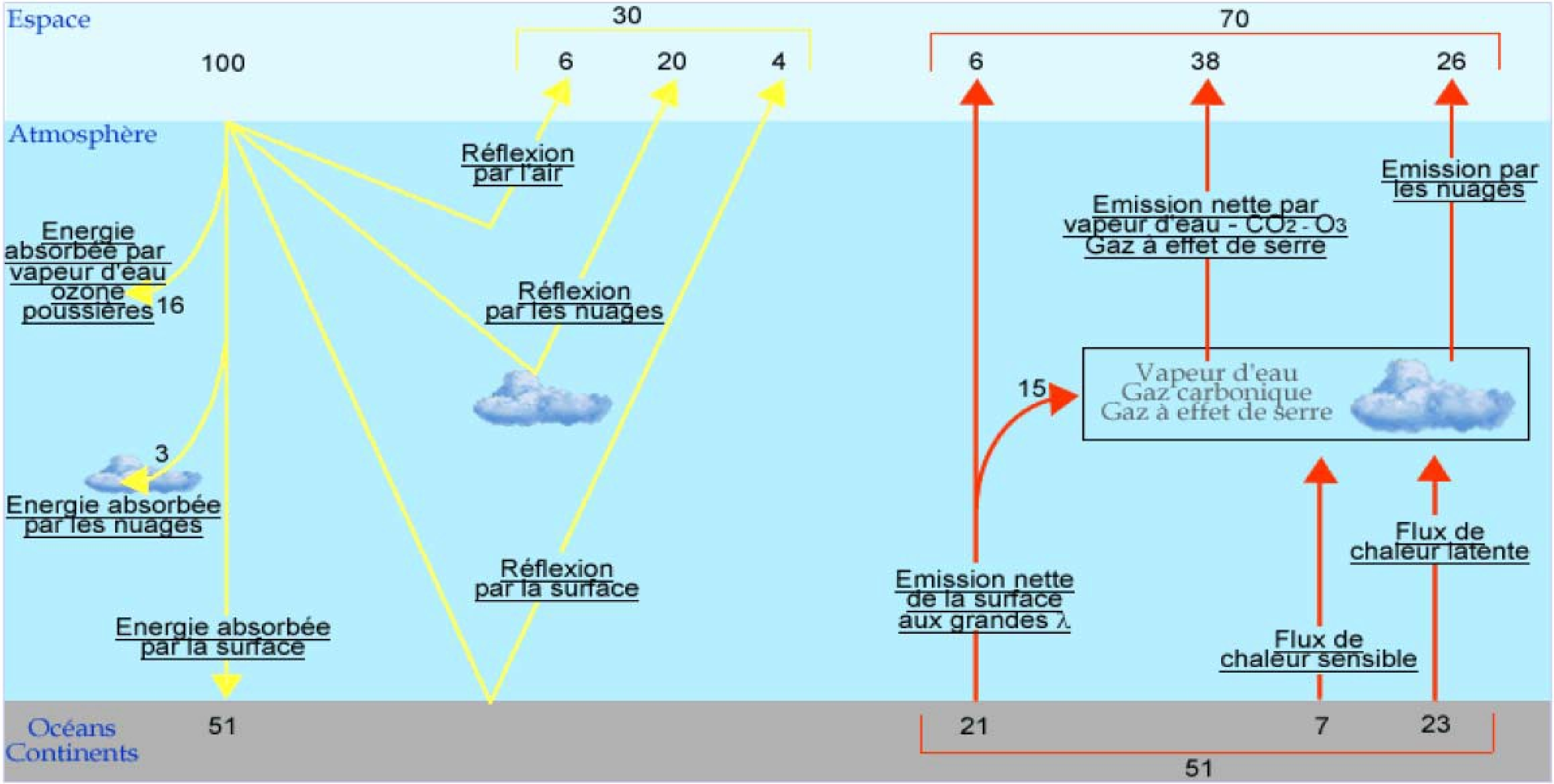
Radiative transfer equation becomes :

$$\frac{1}{\beta_\lambda}\frac{dL_\lambda}{ds} + L_\lambda = J_\lambda$$

Where source function is defined by:

$$J_\lambda = (1 - \omega_\lambda) L_\lambda^0(T) + \frac{1}{4\pi}\omega_\lambda \int_{4\pi} P_\lambda(\vec{\Omega}' \to \vec{\Omega}) L_\lambda(s) d\Omega'$$

Elastic scattering causes angular redistribution of power associated to dipole induced by incoming electric field thanks to particle polarizability. Scattering is playing an important rule for planetary albedo.


Espace
Atmosphère
Océans
Continents
100
30
6
20
4
70
6
38
26
Réflexion par l'air
Energie absorbée par vapeur d'eau ozone poussières
16
Réflexion par les nuages
Emission nette par vapeur d'eau - CO_2 - O_3 Gaz à effet de serre
Emission par les nuages
15
Vapeur d'eau
Gaz carbonique
Gaz à effet de serre
3
Energie absorbée par les nuages
Réflexion par la surface
Emission nette de la surface aux grandes λ
Flux de chaleur latente
Flux de chaleur sensible
Energie absorbée par la surface
51
21
7
23
51

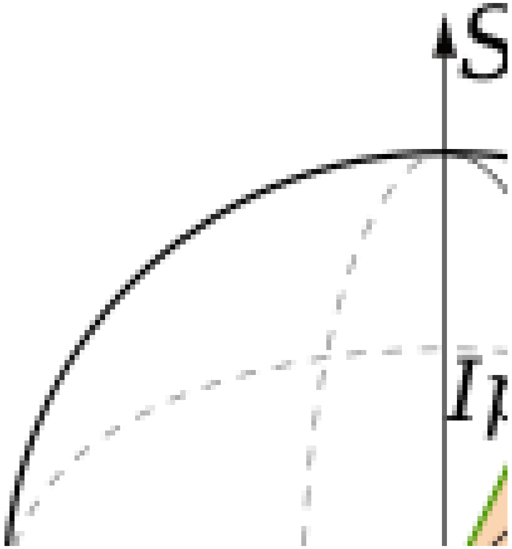
3.4.2 Mueller Matrices[2]

3.4.2.1 Stokes vector

The Stokes parameters are related to the intensity and to the parameters of the ellipse of polarization according to the following relationship:

$$S_0 = I$$

$$S_1 = pI\,\mathrm{co}$$



Sphère de Poincaré représentant les 3 derniers paramètres de Stokes en coordonnées sphériques

Here, pI, 2ψ et 2χ[3] are the spherical coordinates of the 3D vector cartésienens coordinates (S_1,S_2,S_3). I is the total intensity of the beam, and p is the degree of polarization. The four Stokes parameters are often denoted I, Q, U and V, respectively. They are usually combined in a vector called the Stokes vector :

$$\vec{S} = \left(\begin{matrix} S_0 \\ S_1 \end{matrix} \right.$$

The Stokes vector covers non-polarized, partially, or completely polarized light. Jones vector does not cover completely polarized light but is more adapted to the problems involving coherent light. The four Stokes parameters can be easily measured or calculated, hence their usefulness.

---

[2] http://fr.wikipedia.org/wiki/Matrice_de_Mueller

[3] The factor of two before ψ represents the fact that any polarization ellipse is indistinguishable from one rotated by 180°, while the factor of two before χ indicates that an ellipse is indistinguishable from one with the semi-axis lengths swapped accompanied by a 90° rotation.

3.4.2.2 Mueller calculation

Effects an optical system (an air diffusing layer, for example) can be determined by constructing the Stokes vector for the incident light and then applying a calculation to obtain the Mueller Stokes vector of the light to the output of system. And one take into account the polarization change of the radiation due to scattering. The Mueller matrix is a matrix with 4 rows and 4 columns, introduced by Hans Mueller in the 1940*s* are introduced to manipulate the Stokes vector representing the polarization of the incoherent light.

.

$$\vec{S}_o = \mathrm{M}\vec{S}_i \ .$$

3.4.2.3 Unabsorbing isotrope region

- Vacuum, or isotropic and unabsorbing :

$$M = \left(\begin{matrix} 1 \\ 0 \end{matrix} \right.$$

3.4.2.4 Absorbing, isotropic region

- Isotropic region with an absorbing coefficient *1-k* (0<*k*<1) :

$$M = \left(\begin{matrix} k \\ 0 \end{matrix} \right.$$

3.4.2.5 Linear polarizer

- linear polarizer with a transmission angle α :

$$M \quad = 1/2 \left(\begin{matrix} 1 & \cos \\ \cos(2\alpha) & \cos \end{matrix} \right.$$

3.4.2.6 Lame de retard

- Quarter-wave delay plate with azimut α for speed axe :

$$M \quad = \left(\begin{matrix} 1 & 0 \\ 0 & \cos^2(2\alpha) \end{matrix} \right.$$

- Half-wave delay plate with azimut α for speed axe :

$$M \quad = \left(\begin{matrix} 1 & 0 \\ 0 & \cos^2(2\alpha) - \sin^2(2 \end{matrix} \right.$$

3.4.2.7 Phase shifter

$$M = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

4. Implementation to remote sensing

4.1 Layer-by-layer approach

4.1.1 Homogeneous and isothermic medium.

Let be $t_\lambda = t_\lambda(s,0)$ transmission in layer [0 , D], homogeneous (constant concentration and pressure in the overall layer), isothermic, and non-scattering. Then we have :

$$L_\lambda(s) = L_\lambda(0)\, t_\lambda + \int_0^D L_\lambda^0(s')\, \frac{\partial t_\lambda(s,s')}{\partial s'} ds'$$

As temperature T is the same in the overall layer, $L_\lambda^0(s') = L_\lambda^0(T)$

$L_\lambda^0(T)$ is not related to s'. Then we can extract it from integral.

In addition we suppose that :

$$\frac{\partial t_\lambda(s,s')}{\partial s'} ds' = d\, t_\lambda .$$

Integration limits becomes $t_\lambda(s,0) = t_\lambda$ and $t_\lambda(s,s) = 1$. Then we have :

$$L_\lambda(s) = L_\lambda(0)\, t_\lambda + L_\lambda^0(T) \int_{t_\lambda}^1 d\, t_\lambda$$

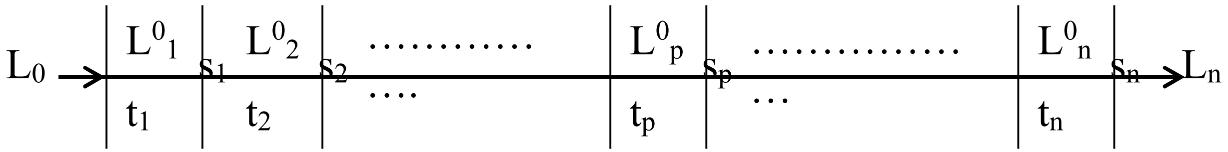
Let be :

$$L_\lambda(s) = L_\lambda(0)\, t_\lambda + L_\lambda^0(T)\left(1 - t_\lambda\right)$$

Radiance outgoing from homogeneous and isothermic layer (having thickness s) is equal to the sum of incoming radiance transmitted to s and radiance emitted by the medium.

4.1.2 Case of several homogeneous and isothermic media.

In order to simulate (and compute) atmospheric radiative transfer, we have to perform a layering of the atmosphere in a succession of homogeneous and isothermal layers as described in the following figure :



We choose the following notations :

- Transmission in the layer i : $t_\lambda(s_{i-1}, s_i) = t_i$
- Blackbody emission at temperature T_i in the layer i : $L^0_\lambda(T_i) = L^0_i$
- Radiance outgoing from layer i : $L_\lambda(s_i) = L_i$

We are looking for the radiance that outcomes from layer n, knowing that a radiance L_0 incomes in the first layer.

Taking results obtained previously for a single homogeneous and isothermal layer, we can apply it to the first layer using the notations defined above :

$$L_1 = L_0\, t_1 + L^0_1 (1 - t_1)$$

$$\begin{aligned} L_2 &= L_1\, t_2 + L^0_2 (1 - t_2) \\ &= \left[L_0\, t_1 + L^0_1 (1 - t_1)\right] t_2 + L^0_2 (1 - t_2) \\ &= L_0\, t_1 t_2 + L^0_1 (1 - t_1) t_2 + L^0_2 (1 - t_2) \end{aligned}$$

$$
\begin{aligned}
L_3 &= L_2\, t_3 + L_3^0\left(1 - t_3\right) = \left[L_0\, t_1 t_2 + L_1^0\left(1 - t_1\right)t_2 + L_2^0\left(1 - t_2\right)\right] t_3 + L_3^0\left(1 - t_3\right) \\
&= L_0\, t_1 t_2\, t_3 + L_1^0\left(1 - t_1\right)t_2\, t_3 + L_2^0\left(1 - t_2\right)t_3 + L_3^0\left(1 - t_3\right)
\end{aligned}
$$

To find a simple formulation of out coming radiance from layer, we can introduce the following notations :

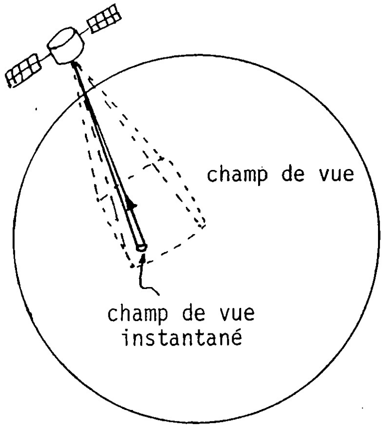
$$\zeta_p = \prod_{k=p+1}^{n} t_k \quad p = 1 \text{ à } n\text{-}1$$

$$\zeta_n = 1$$

$$
\begin{aligned}
L_n &= L_0\, \xi_0 + L_1^0\left(1 - t_1\right)\xi_1 + L_2^0\left(1 - t_2\right)\xi_2 + \ldots + L_{n-1}^0\left(1 - t_{n-1}\right)\xi_{n-1} + L_n^0\left(1 - t_n\right)\xi_n \\
&= L_0\, \xi_0 + \sum_{p=1}^{n} L_p^0\left(1 - t_p\right)\xi_p \\
&= L_0\, \xi_0 + \sum_{p=1}^{n} L_p^0\left(\xi_p - \xi_{p+1}\right)
\end{aligned}
$$

We find again a two terms expression : incoming radiance transmitted to s_n and radiance emitted by each layer and transmitted to s_n.

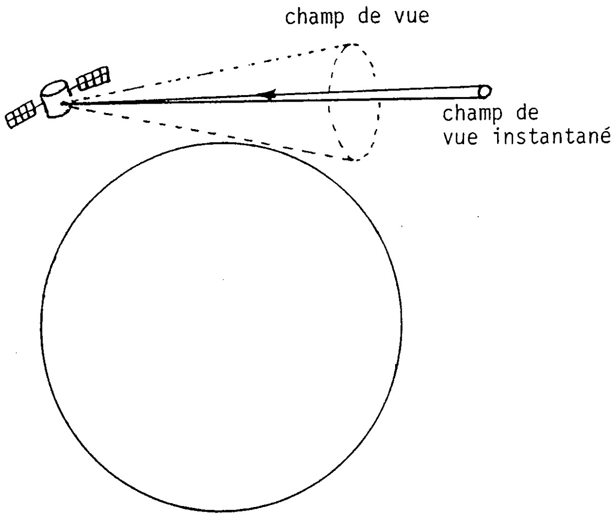
4.2 Line of sight geometries



Visée au nadir

Bonne résolution horizontale
Médiocre résolution verticale

2 modes
- émission (thermique ou propre)
- rétrodiffusion solaire

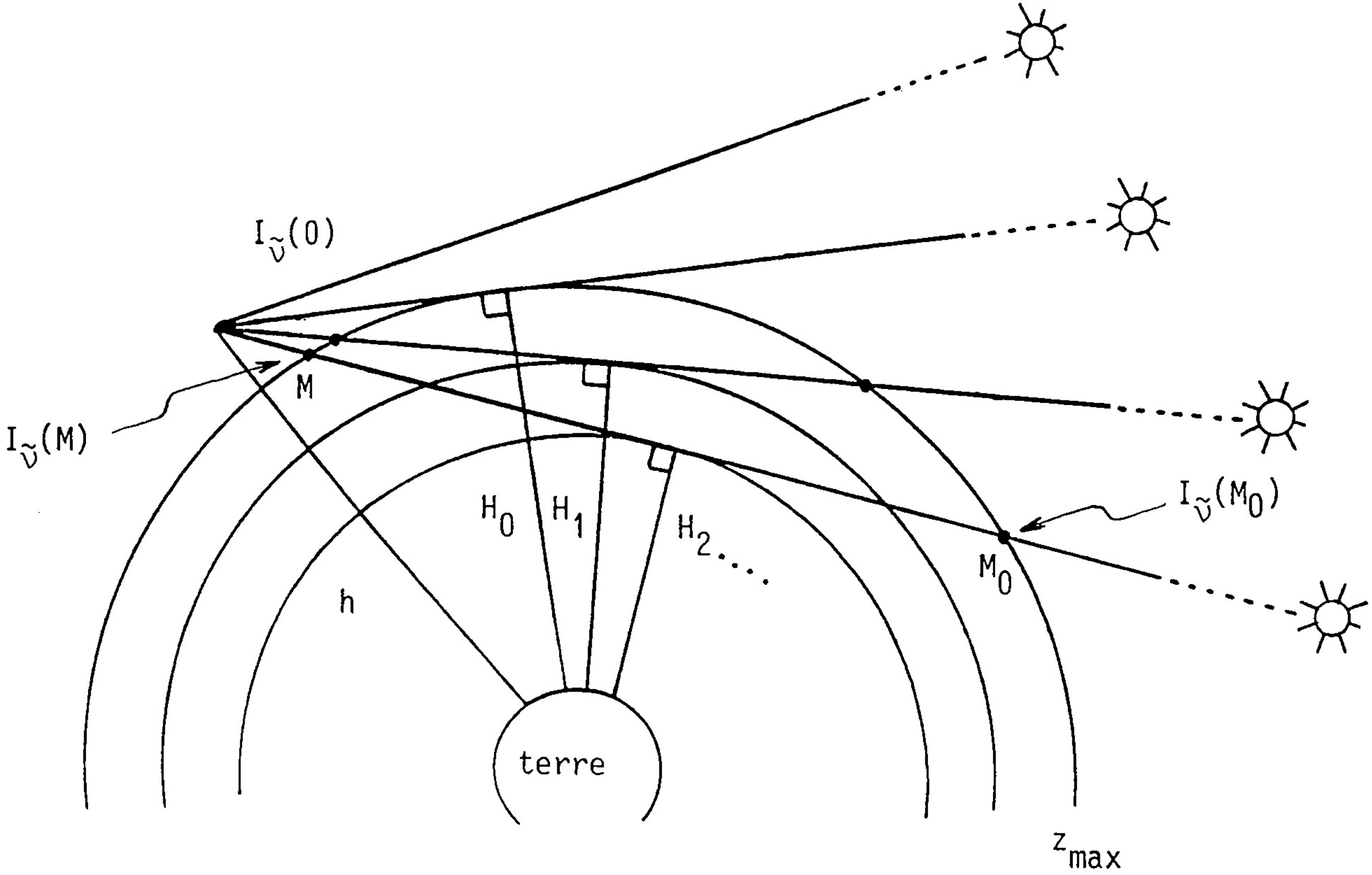


Visée au limbe

Bonne résolution verticale
Médiocre résolution horizontale

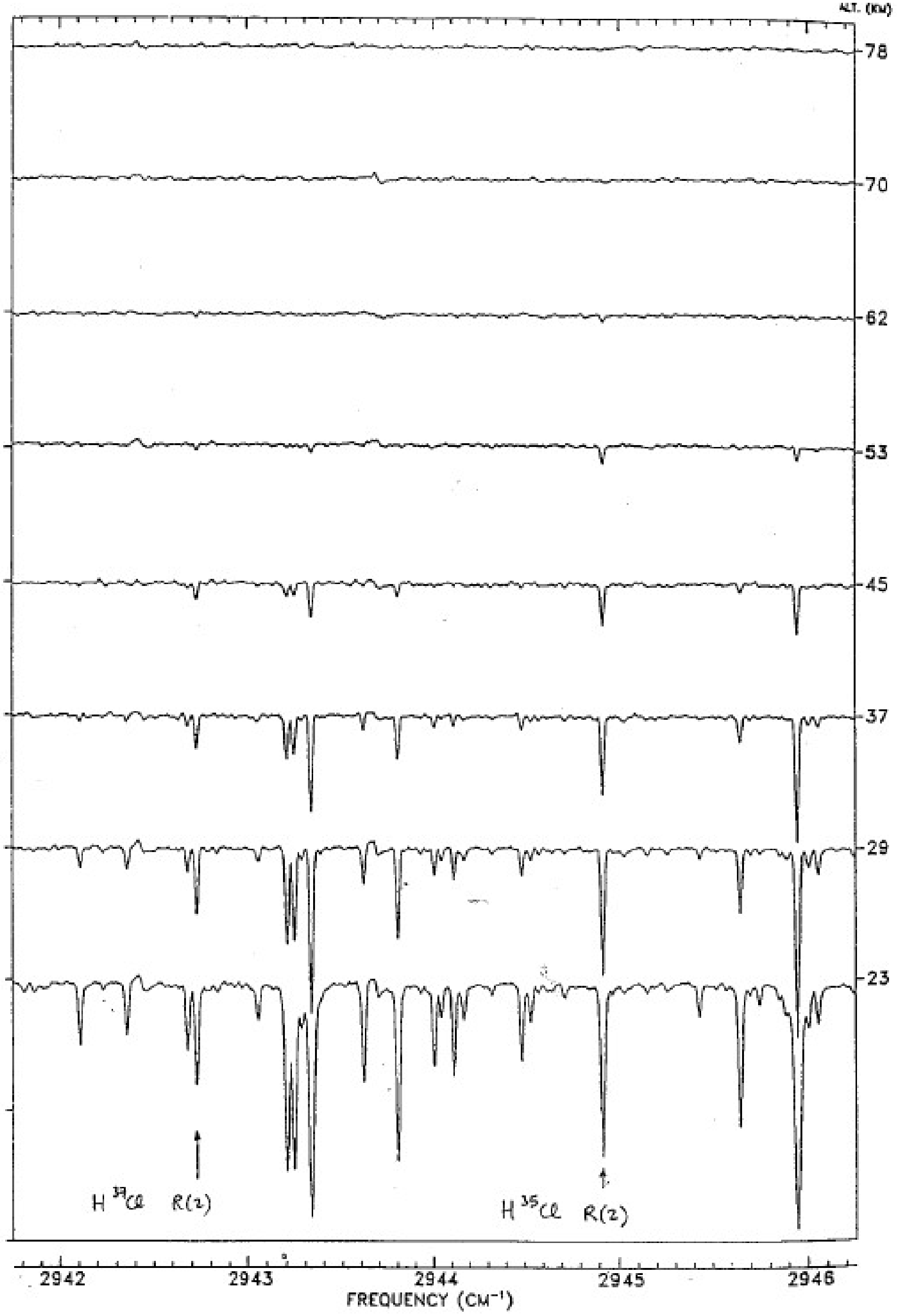
2 modes
- émission (thermique ou propre)
- occultation solaire

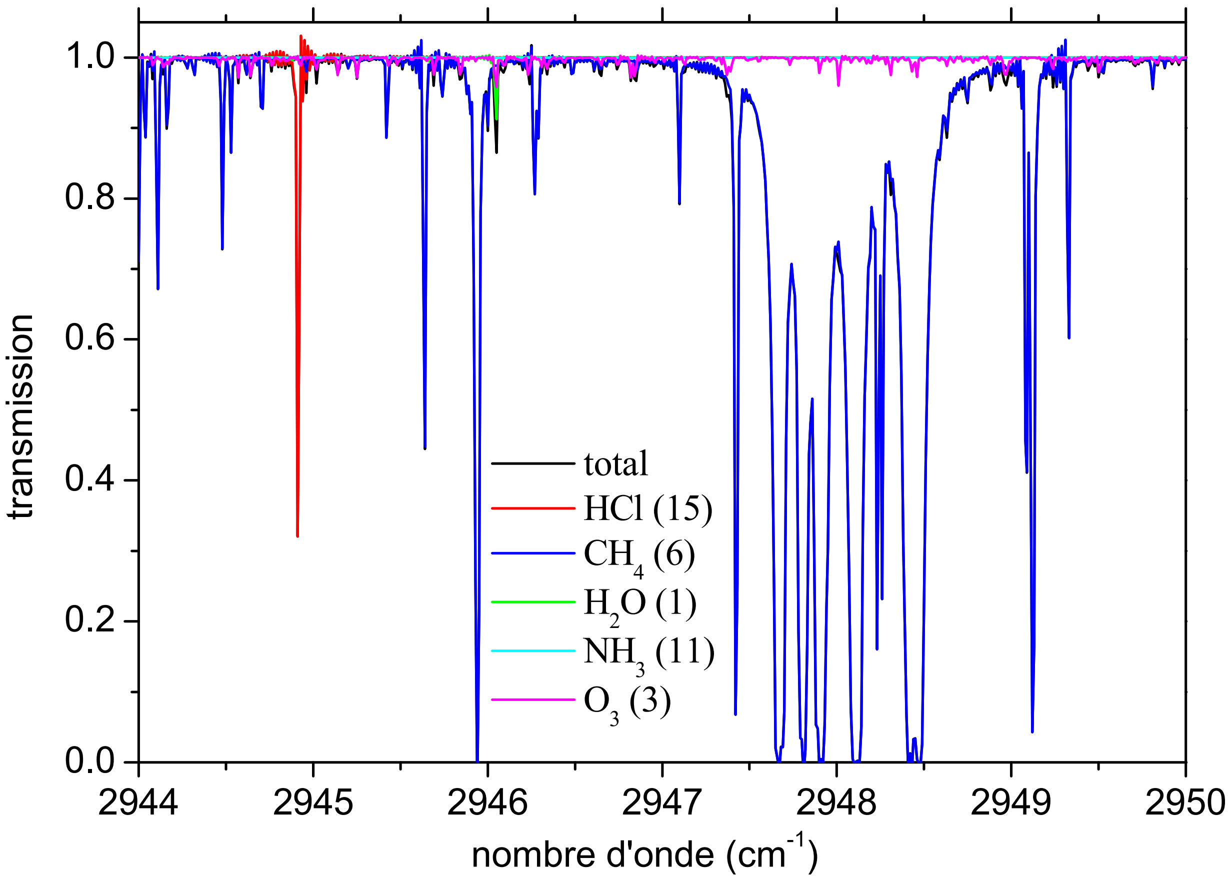
- limb measurements: good vertical resolution, horizontal resolution ~ 400 km. sounding above ~ 6 km.
- Nadir measurements : good horizontal resolution (10 x 10 km), poor vertical resolution. Tropospheric and/or stratospheric sounding (depending on species, instrument capabilities, …) with or without differentiations of the two columns.
- Upward measurements (cf. nadir)
- backscattering : typical of improved nadir, or same as limb (depend on geometries).



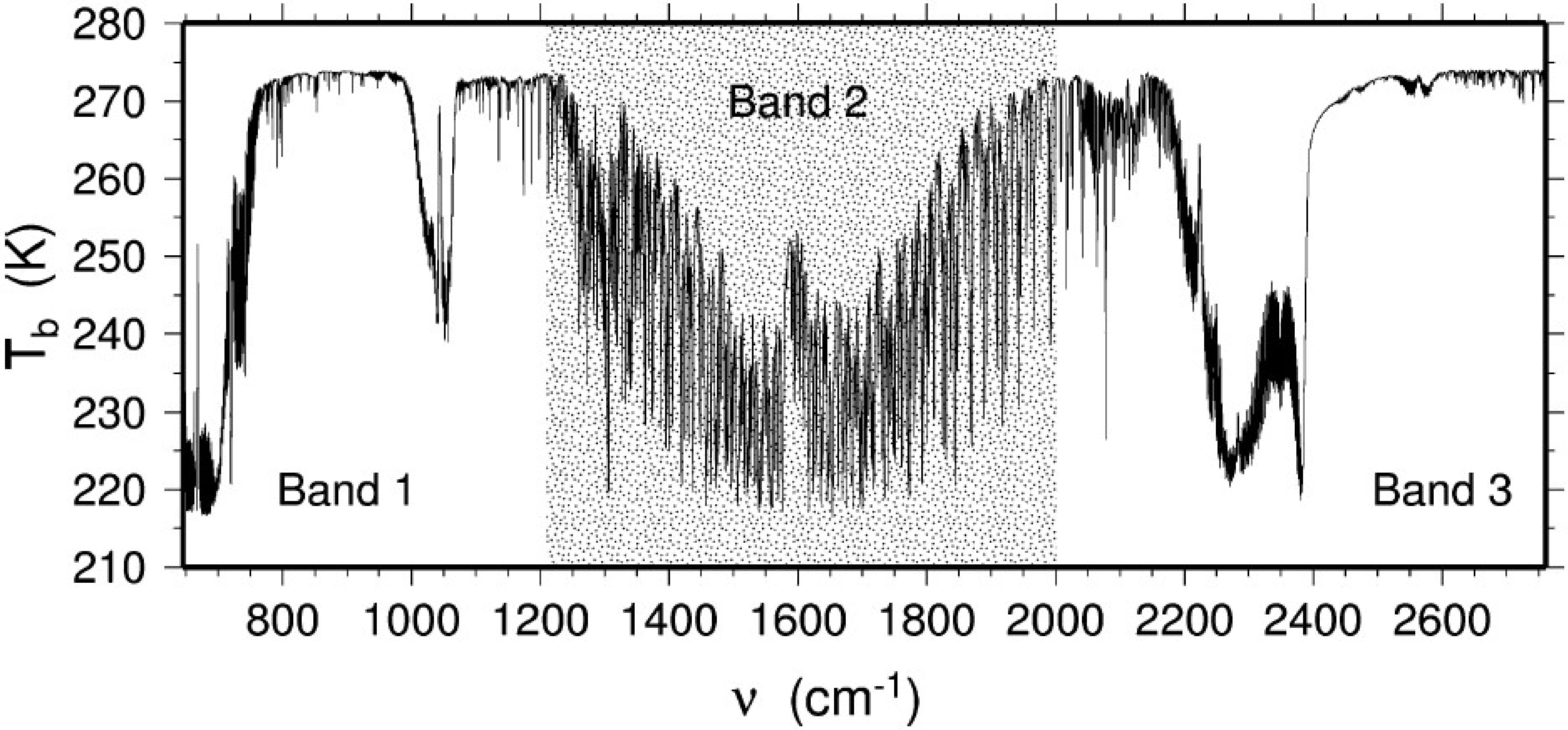
Geometry for a solar occultation mode absorption measurement.

Example of limb spectra : ATMOS experiment.





Example of nadir spectra : IASI



[4]

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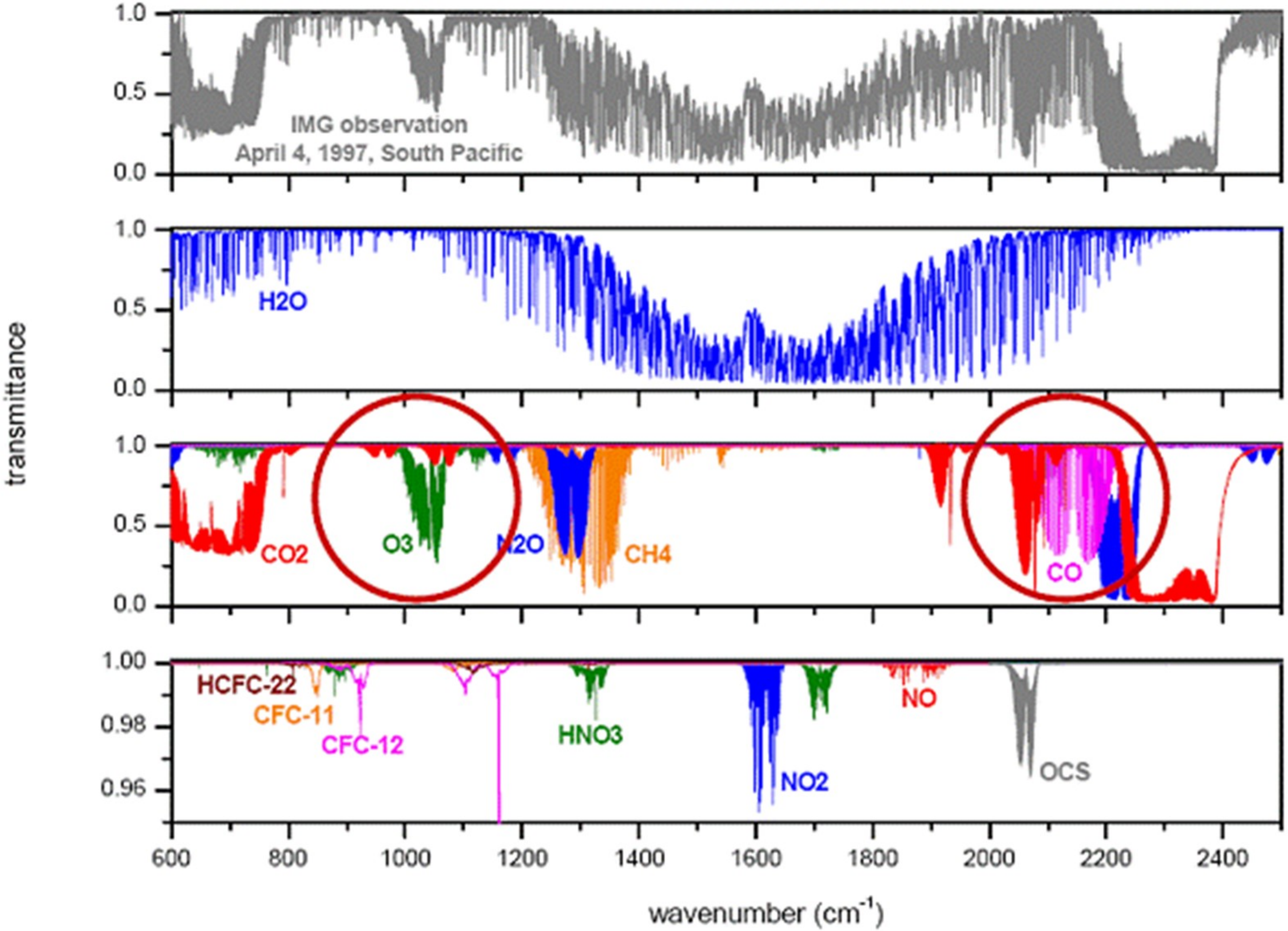
[4] Peter Schlüssel *, Tim H. Hultberg, Pepe L. Phillips, Thomas August, Xavier Calbet, The operational IASI Level 2 processor , Advances in Space Research 36 (2005) 982–988


1.0
0.5
0.0
IMG observation
April 4, 1997, South Pacific
1.0
0.5
0.0
H2O
1.0
0.5
0.0
CO2
O3
N2O
CH4
CO
1.00
0.98
0.96
HCFC-22
CFC-11
CFC-12
HNO3
NO2
NO
OCS
transmittance
600
800
1000
1200
1400
1600
1800
2000
2200
2400
wavenumber (cm^{-1})

4.3 Case study

We look for to infer NO_2 vertical profile in Earth stratosphere from infrared high spectral resolution measurements. For that we use spectra recorded by a FTS interferometer looking the sun through the atmosphere from a satellite platform.

1 - Line of sight geometry

In order to solve the inverse problem, we first cut atmosphere in layer shaving a 5 km thickness (figure 1). We consider a limb optical path having a tangent altitude of 25 km.

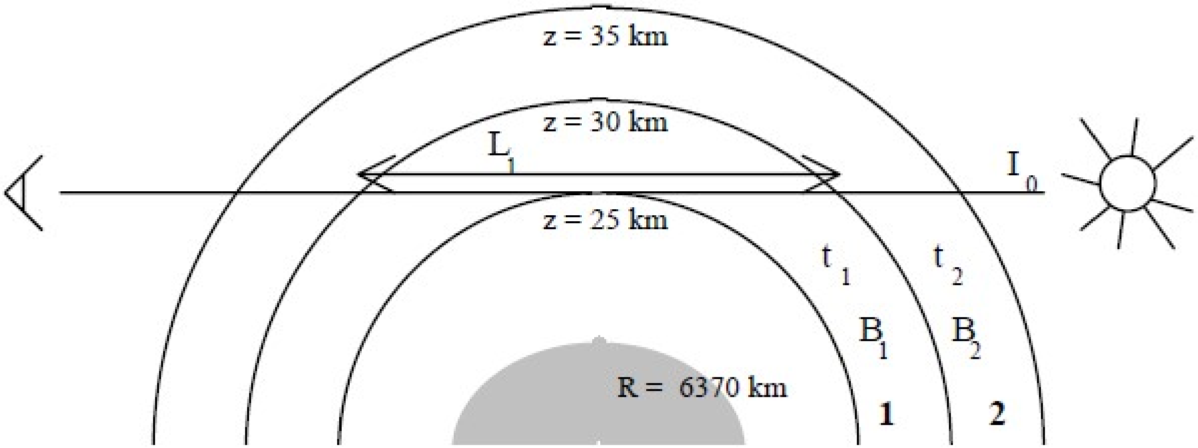


Figure 1

For the calculation of the radiation path length in each layer, the refraction will be neglected.. What is the length L_1 and L_2 of the path in the two considered layers supposing that the lower altitude of first layer is 25 km ?

- Earth radius = 6370 km

<u>**2** – radiative transfer equation.</u>

2.a We suppose that temperature, pressure, and gas concentrations of absorbing gas are constant in each considered layer. Starting from the general radiative transfer equation

$$I_{\tilde{\nu}}(s) = I_{\tilde{\nu}}(0)\,t_{\tilde{\nu}}(s,0) + \int_0^s J_{\tilde{\nu}}(s')\frac{\partial t_{\tilde{\nu}}(s,s')}{\partial s'}\,ds'$$

express simply radiance outcoming from the atmosphere as a function of transmission in each layer (t_1, t_2), of Planck function in each layer (B_1, B_2), and of incoming radiance at the top of the atmosphere I_0 (we will suppose atmosphere at the local thermodynamic equilibrium).

2.b. In a solar occultation mode scenario, we suppose as negligible the inner emission term in the atmosphere in the radiative transfer equation. Demonstrate that we obtain a simple expression of the transmission along the line of sight as a function of t_1 and t_2 if the solar radiance at the top of the atmosphere is known.

<u>**3** – Optical depth calculation.</u>

We look to infer NO_2 profile from an isolated line of this species around 2914 cm^{-1} (see figure 2).

Calculate the Doppler half-with at half maximum in the tangent layer. Can you confirm that a Doppler profile can be used here ?

Calculate value of line at the top of the profile (supposed to be Doppler)

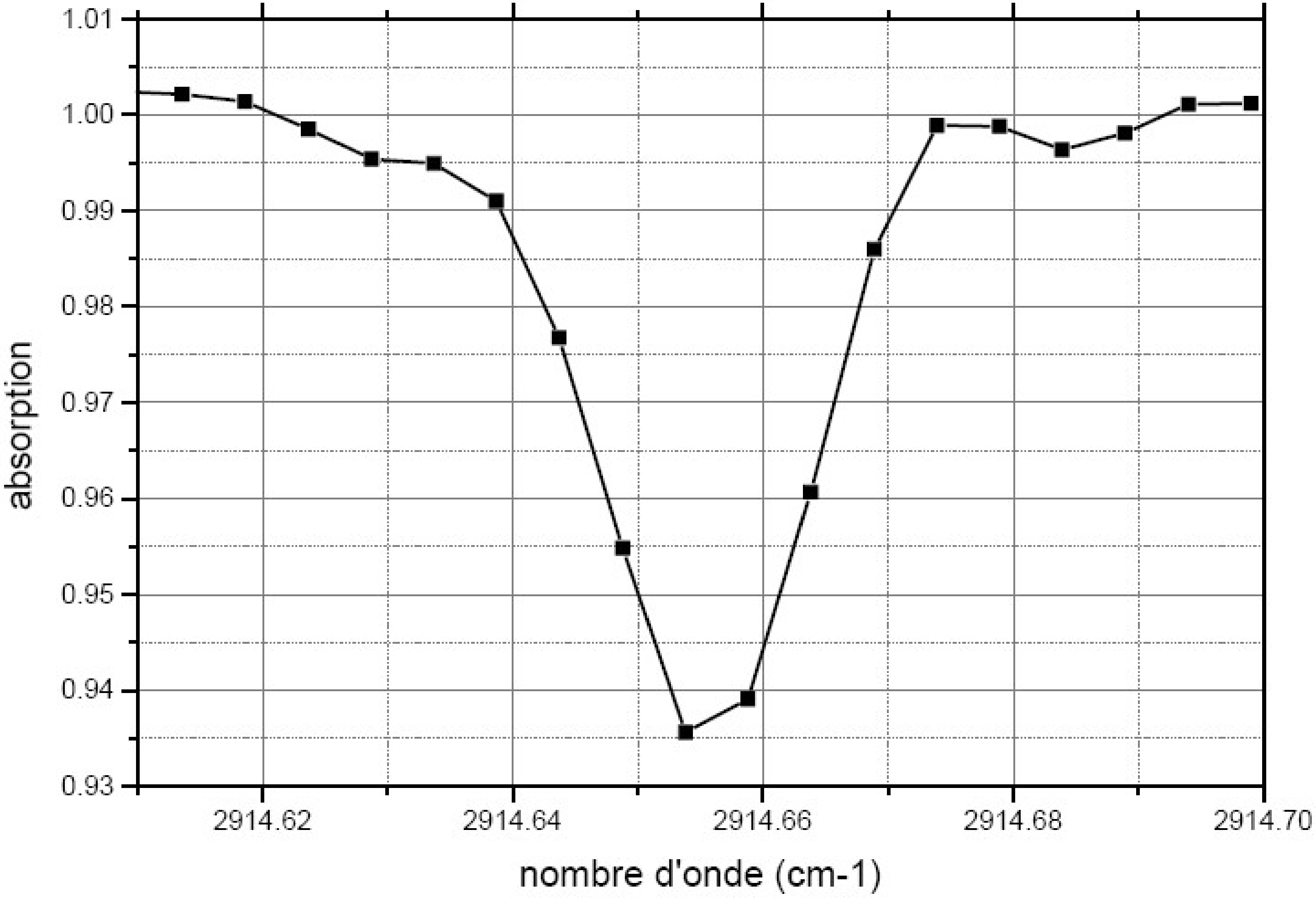
Calculate value of optical depth at wavenumber $\widetilde{\nu}=\widetilde{\nu}_0$ (top of the line) in the tangent layer.

If we consider only the tangent layer in the radiative transfer equation, show from preceding results that the equivalent width S (cf. equation below) of the considered line can be expressed as follow in the hypothesis of small absorption :

$$S = \int_0^{+\infty} \left(1 - t_{\widetilde{\nu}}\right) d\widetilde{\nu} = k^N_{\widetilde{\nu}_0} \, N \, r \, L_1$$

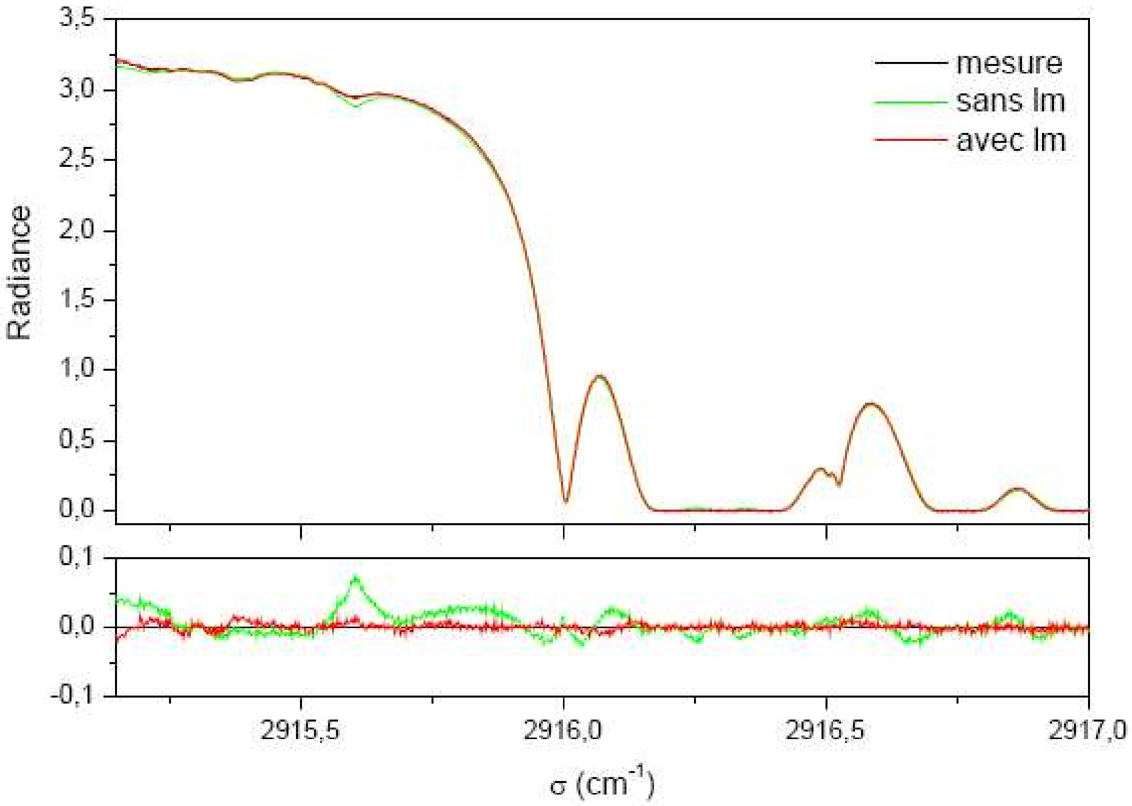
Confirm dimension homogeneity of equivalent width S.

- line intensity : $k^N_{\widetilde{\nu}_0} = 6.07 \text{x} 10^{-21} \text{cm}^{-1} / (\text{molecule.cm}^{-2})$
- line position: $\widetilde{\nu}_0 = 2914.64 \text{ cm}^{-1}$
- air broadening coefficient : $\gamma_0^L = 0.0063 \text{ cm}^{-1}.\text{atm}^{-1}$
- a priori NO_2 volume mixing ratio around 25 km altitude: r = 1 ppbv
- Temperature in the tangent layer : T = 200 K
- Pressure in the tangent layer : P = 18.3 hPa
- reference temperature : $T_0 =$ 273.15 K
- reference pressure : 1013.25 hPa
- Lochmidt number : £ = $2.686763 \text{x} 10^{19}$ molecule.cm^{-3}



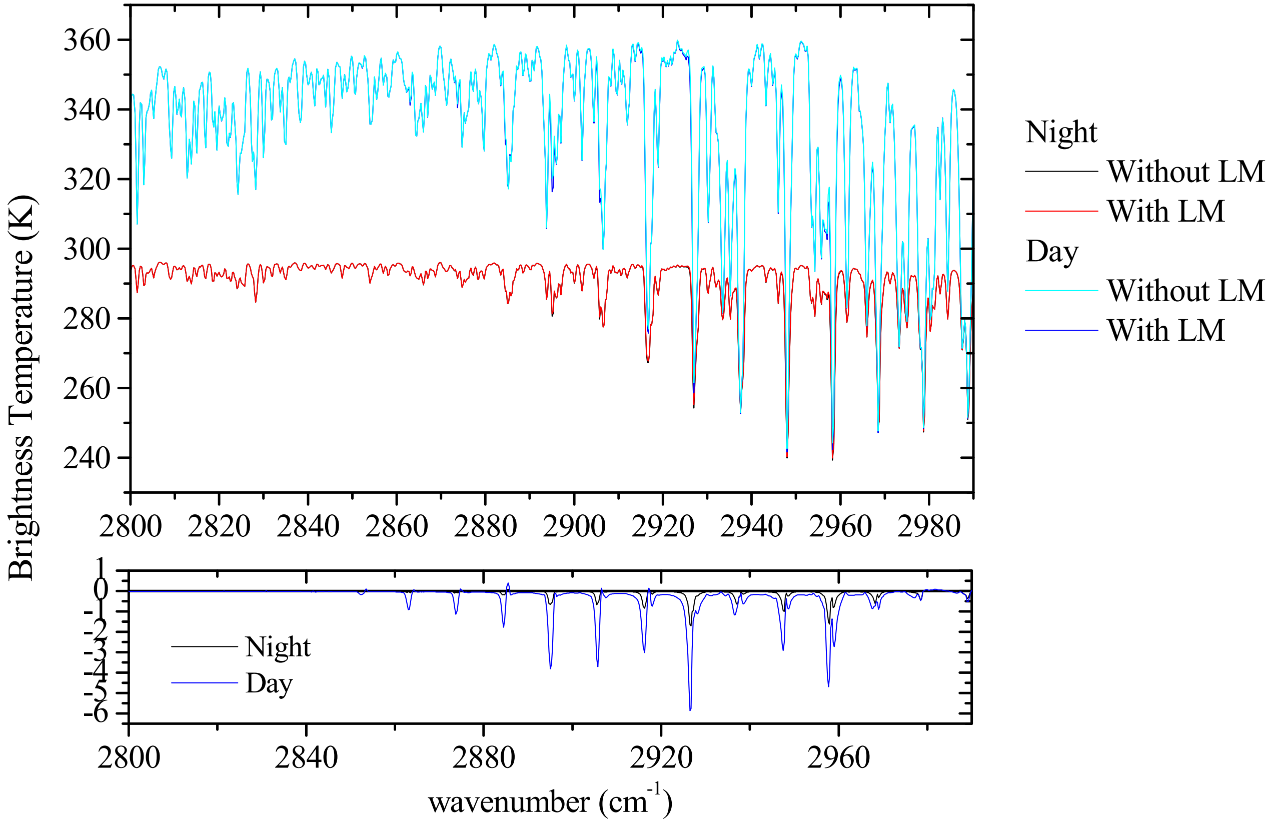
4 - NO$_2$ concentration calculation.

We suppose that tangent layer only contributes significantly to absorption. Using the above spectrum (figure 2), infer equivalent width S of the NO$_2$ line, and then infer the volume mixing ratio of NO$_2$ in the tangent layer.



5. Outcomes

- refraction
- NLTE
- Chemistry and photochemistry
- Clouds and aerosols
- Line shapes :
    - Narrowing
    - Line mixing
- Solar spectrum
- Polarization
- statistical approaches / neural networks



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Physique moléculaire, physique de l'atmosphère: comptes rendus de l'Ecole d'Hiver Applications de la Physique Moléculaire a l'Atmosphère et l'Environnement ; Montfoulon (pres d'Alençon), 1 - 10 Dec. 1980, Camy-Peyret C., Amat G, Paris. Ed. du Centre Nat. de la Recherche Scientifique. 1982.