MHD Turbulence in Partially Ionized Gas

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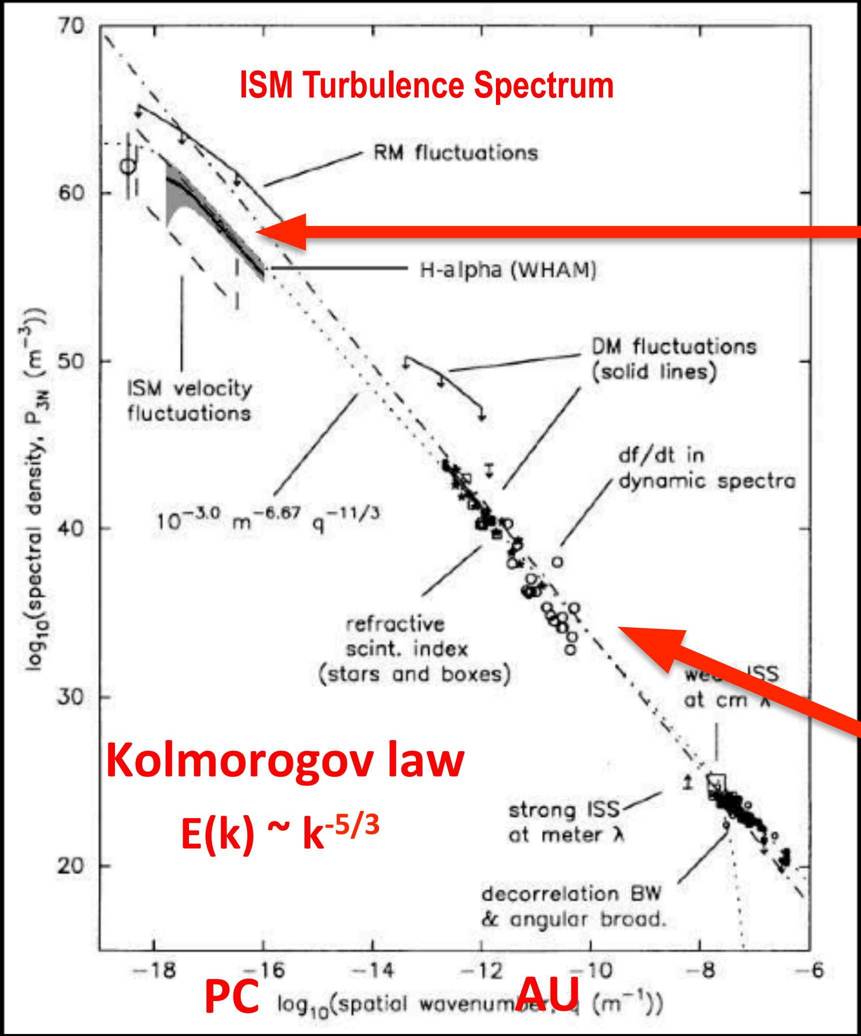
The interstellar medium is

turbulent, magnetized, and partially ionized.

Big power law in the WIM

Turbulent power density



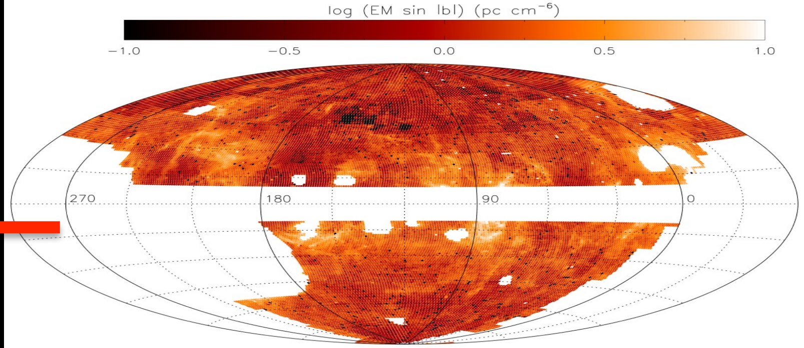
ISM Turbulence Spectrum

Kolmorogov law

$E(k) \sim k^{-5/3}$

PC

AU



WHAM emission: density fluctuations

Haffner et al. 2003

Interstellar scattering of nearby pulsars

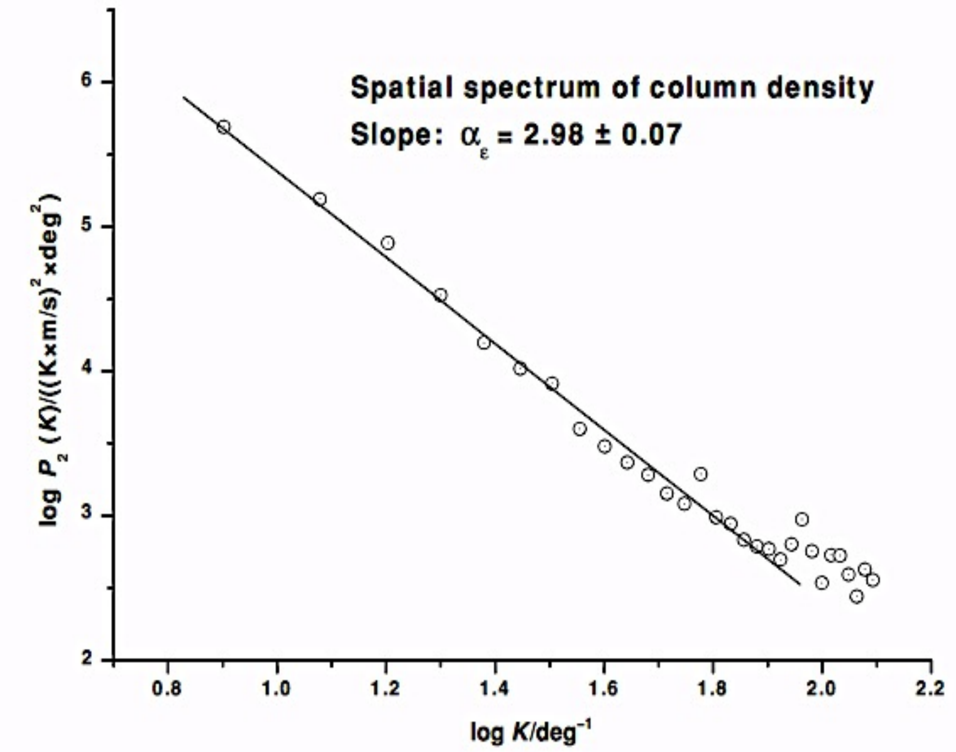
Armstrong et al. 1995, Chepurnov & Lazarian 2009

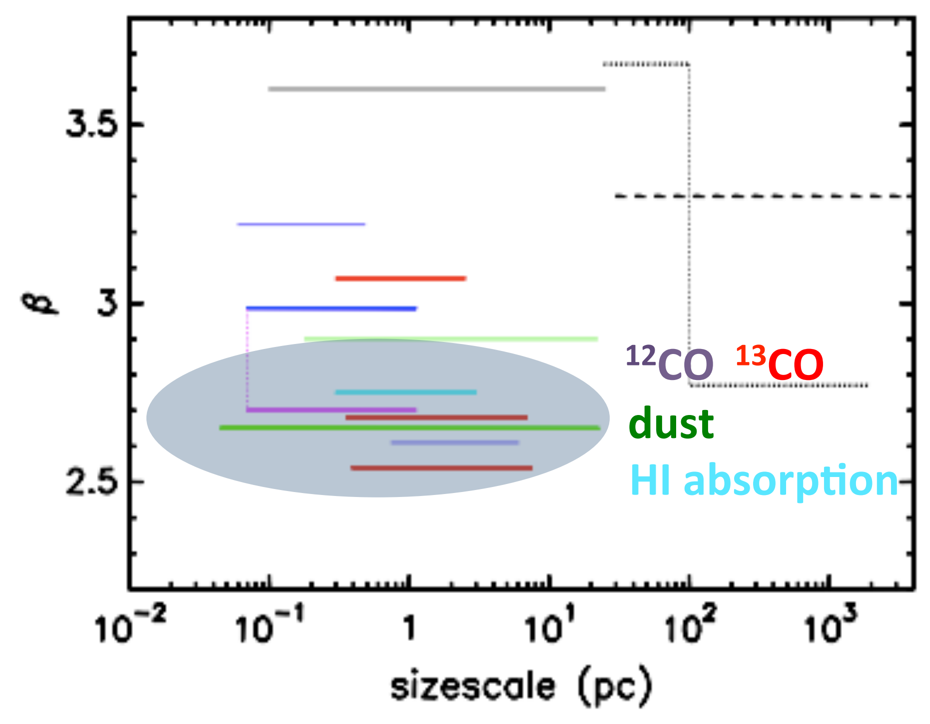
WNM

CNM & MC

Density spectrum $\beta = 2.98$

$\beta < 3$





Chepurnov et al. 2010

Hennebelle & Falgarone 2012

mildly supersonic

highly supersonic

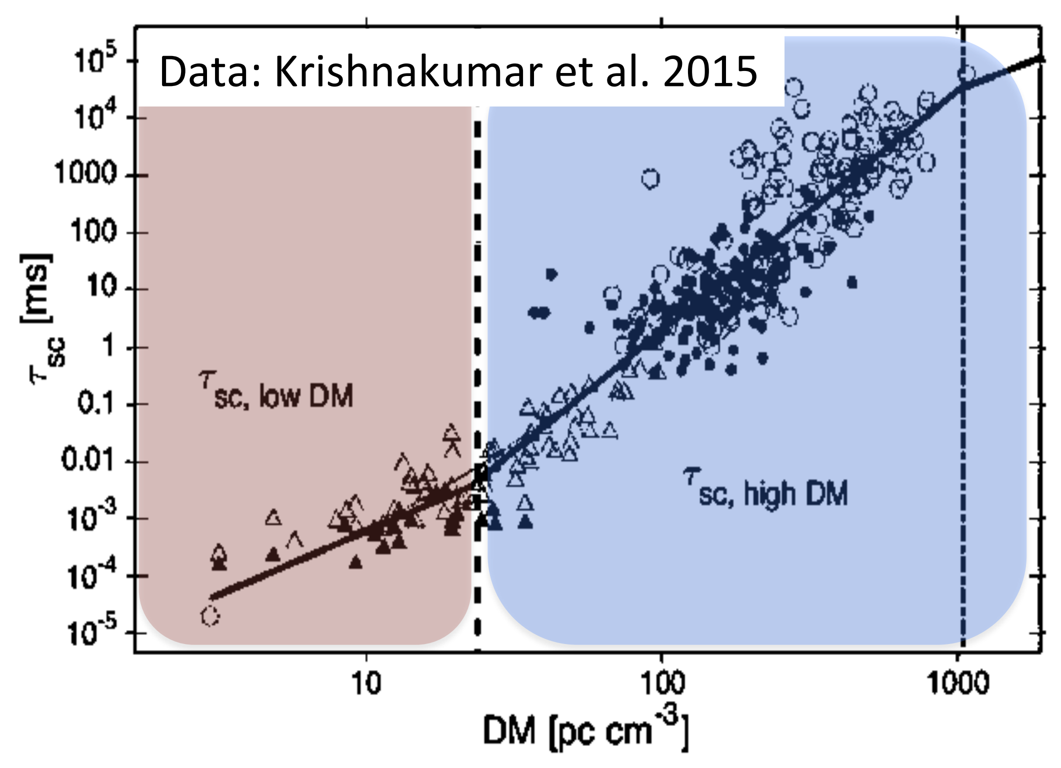
Radio diagnostic of interstellar electron density fluctuations

WIM Kolmorogov

CNM & MC highly supersonic

β = 11/3

β = 2.6



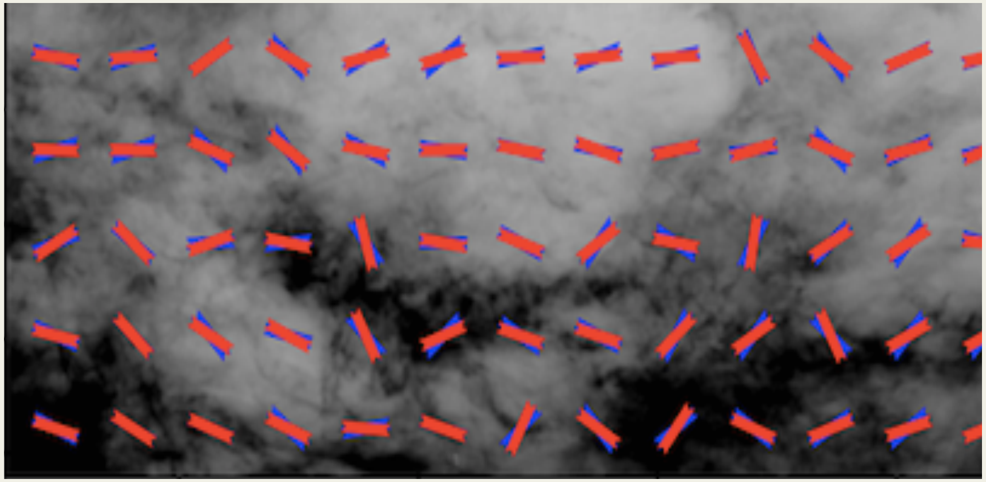
Data: Krishnakumar et al. 2015

Nearby / high-latitude pulsars

Distant & low-latitude pulsars

Xu & Zhang 2017 ApJ, 835, 2

Turbulent, magnetized, and partially ionized interstellar medium



HI traces magnetic fields
(Alex Lazarian's talk)

Physical processes in MCs

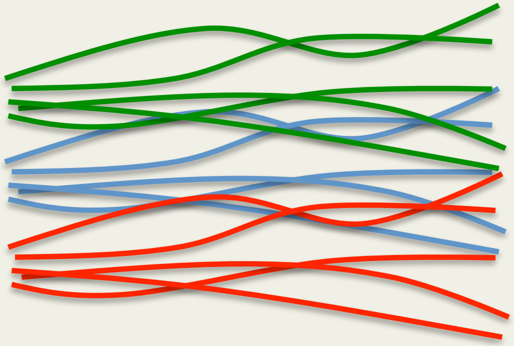
MHD turbulence in a partially ionized medium

Alfven (GS95)   Slow   Fast

GS95 model for Alfvenic turbulence

Goldreich & Sridhar 1995

Local mean magnetic field

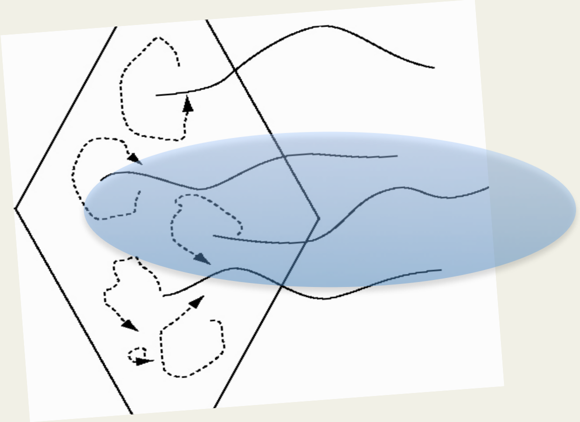


$\|$ Linear Alfven waves
in one wave packet

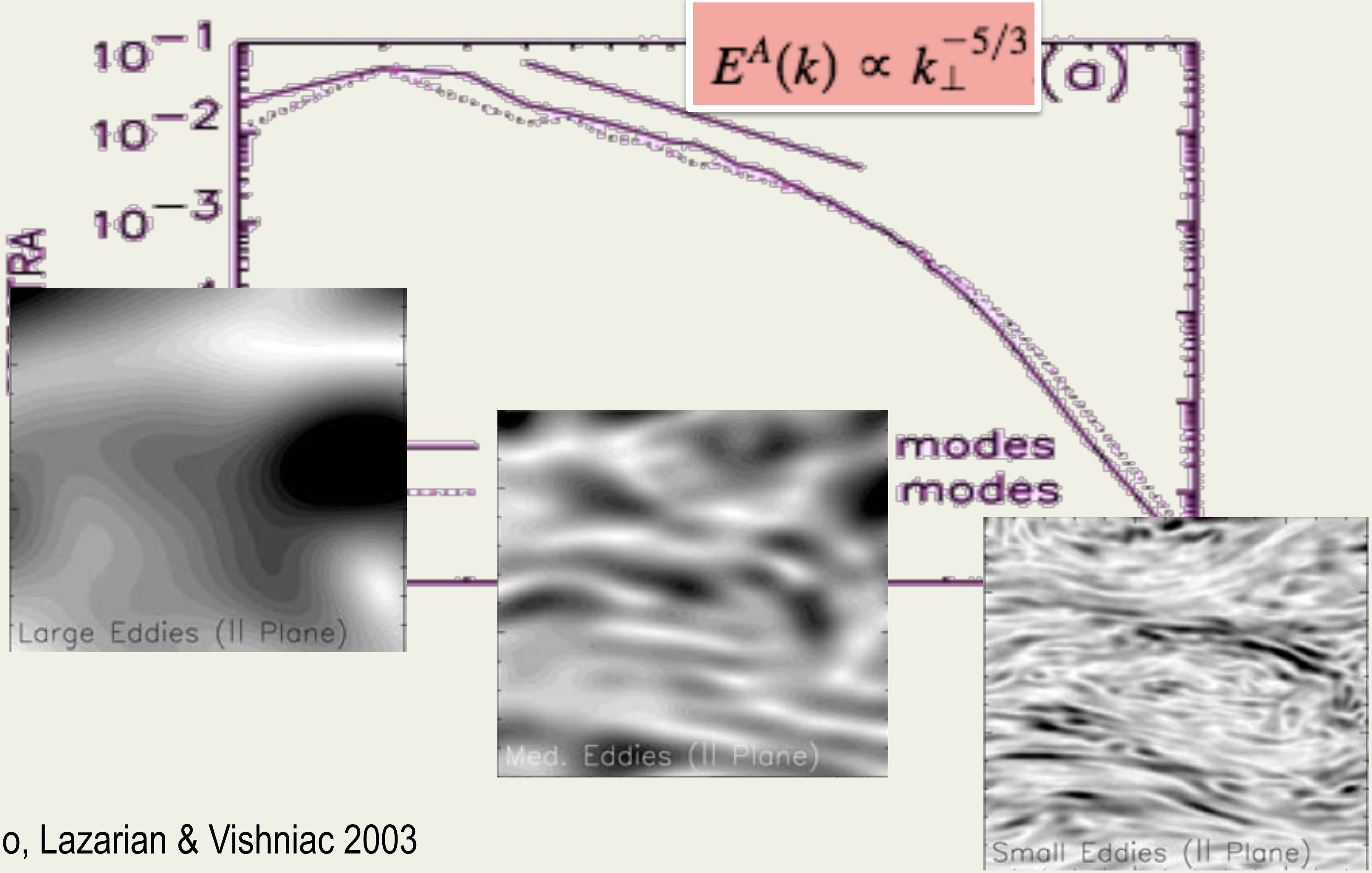
Nonlinear interaction → $\perp$ Nonlinear cascade

Critical balance

$$\boxed{k_\perp \mathrm{v}_\perp = k_\| \mathrm{V}_\mathrm{A}}$$



GS95 model for Alfvenic turbulence



$$E^A(k) \propto k_\perp^{-5/3}$$

Cho, Lazarian & Vishniac 2003

Anisotropy is larger at smaller scales

MHD waves in partially ionized gas

Equations governing the linear MHD waves:

$$\rho_{\mathrm{i}} \frac{\partial \mathbf{v}_{\mathrm{i}}}{\partial t} = -\nabla p_{\mathrm{i}} + \frac{1}{\mu} (\nabla \times \mathbf{b}) \times \mathbf{B} - \alpha_{\mathrm{in}} (\mathbf{v}_{\mathrm{i}} - \mathbf{v}_{\mathrm{n}}) \,,$$

$$\rho_{\mathrm{n}} \frac{\partial \mathbf{v}_{\mathrm{n}}}{\partial t} = -\nabla p_{\mathrm{n}} - \alpha_{\mathrm{in}} (\mathbf{v}_{\mathrm{n}} - \mathbf{v}_{\mathrm{i}}) \,,$$

partial ionization

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{v}_{\mathrm{i}} \times \mathbf{B}) \,,$$

$$\frac{\partial p_{\mathrm{i}}}{\partial t} = -\gamma P_{\mathrm{i}} \nabla \cdot \mathbf{v}_{\mathrm{i}},$$

$$\frac{\partial p_{\mathrm{n}}}{\partial t} = -\gamma P_{\mathrm{n}} \nabla \cdot \mathbf{v}_{\mathrm{n}},$$

e.g. Balsara 1996

Alfven waves in partially ionized gas

Dispersion relations:

$$\omega^3 + i(\tau_v^{-1} + (1+\chi)\nu_{ni})\omega^2 - (k^2\cos^2\theta V_{Ai}^2 + \chi\tau_v^{-1}\nu_{ni})\omega - i(\tau_v^{-1} + \nu_{ni})k^2\cos^2\theta V_{Ai}^2 = 0.$$

neutral-neutral collision frequency

neutral-ion collision frequency

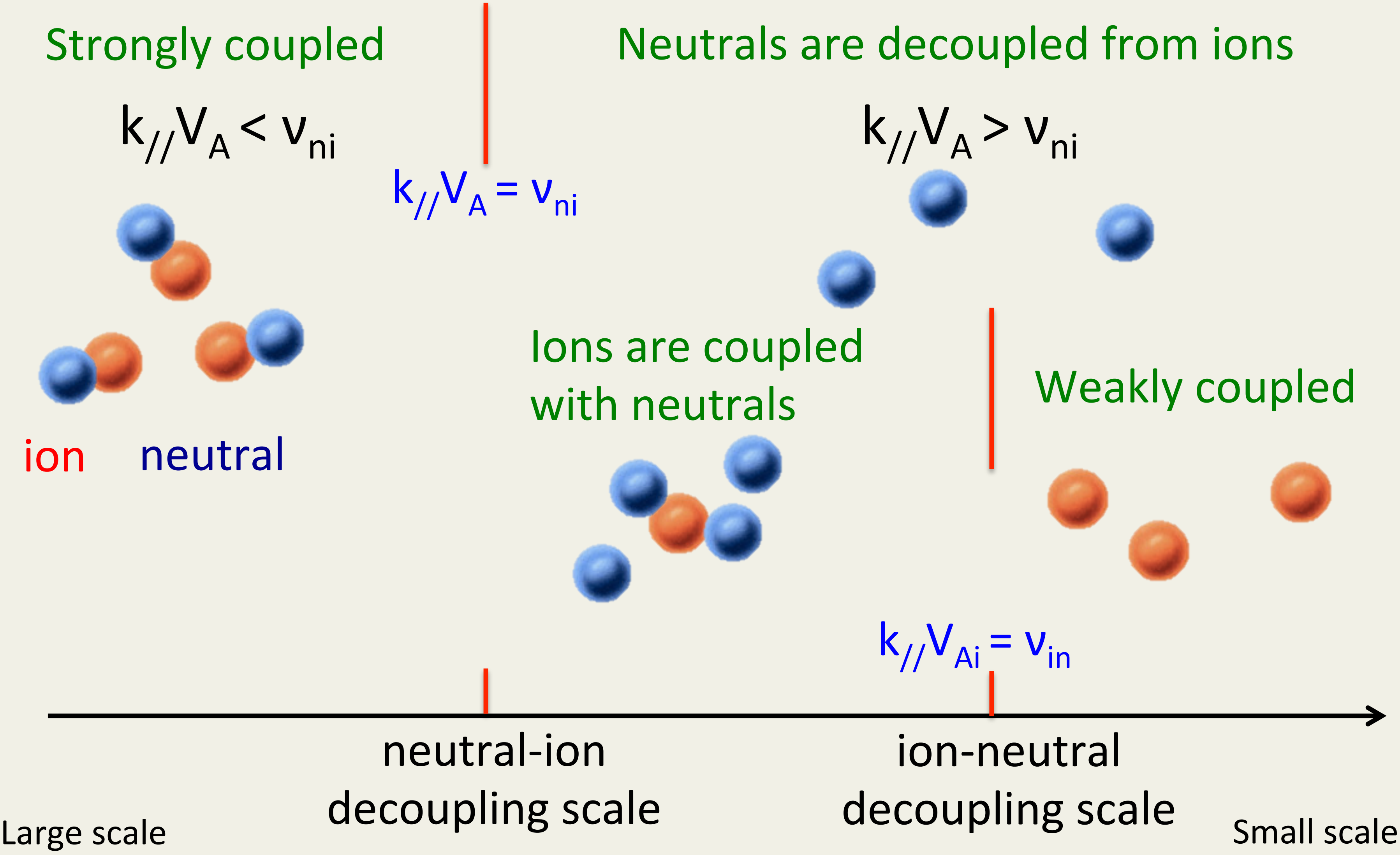
$$\omega = \omega_R + i\,\omega_I$$

Propagation

Damping: ion-neutral collisions
neutral viscosity

Xu et al. 2015, ApJ, 810, 44; 2016, ApJ, 826, 166

Alfven waves in a partially ionized gas



Strongly coupled

$k_{//}V_A < \nu_{ni}$

$k_{//}V_A = \nu_{ni}$

Neutrals are decoupled from ions

$k_{//}V_A > \nu_{ni}$

Ions are coupled with neutrals

Weakly coupled

ion neutral

$k_{//}V_{Ai} = \nu_{in}$

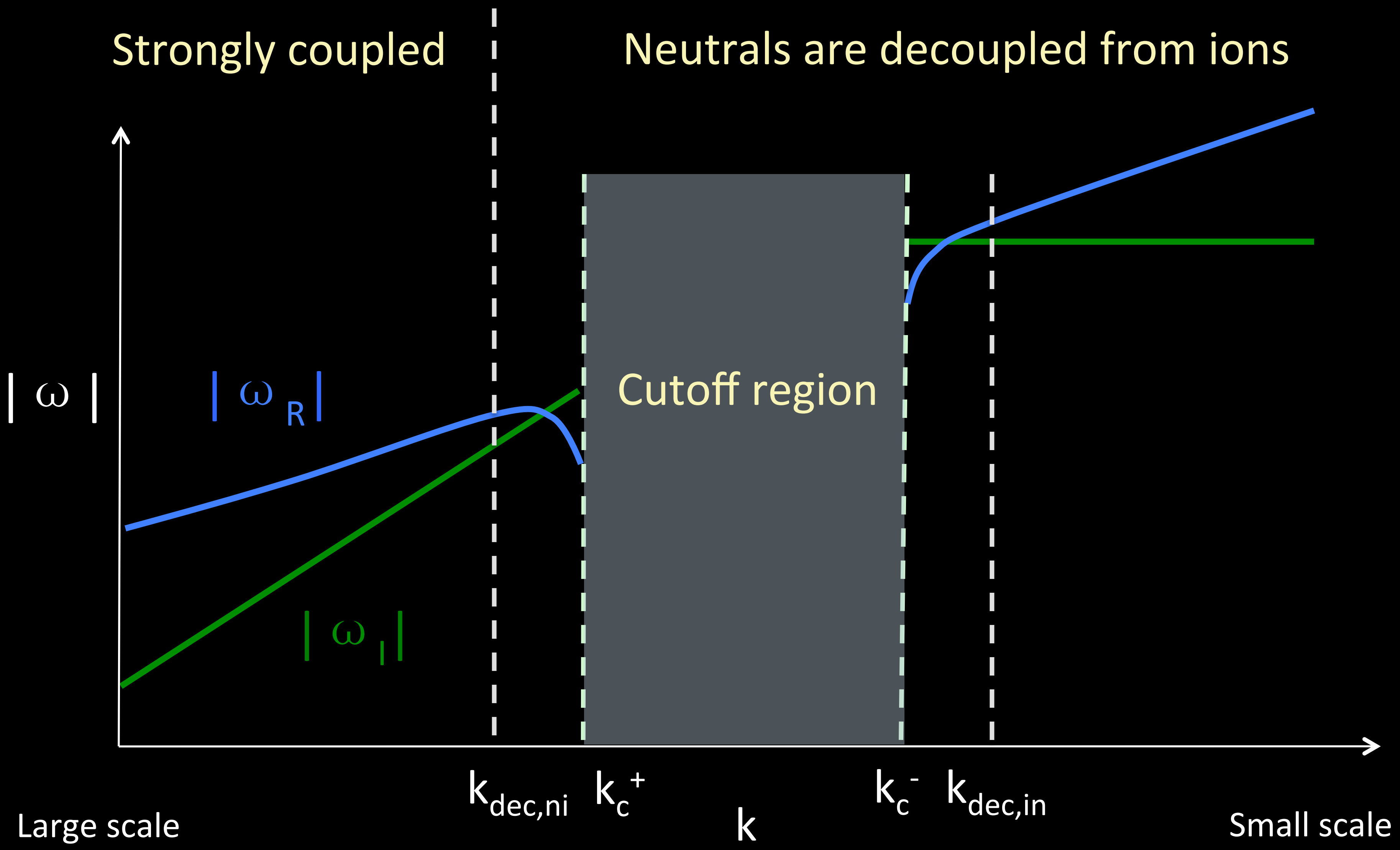
neutral-ion decoupling scale

ion-neutral decoupling scale

Large scale

Small scale

Alfven waves in a partially ionized gas



Cutoff of Alfven waves

$$|\omega_R| = k_\parallel V_A = |\omega_I|$$

Damping of Alfvenic turbulence

$$\boldsymbol{\tau_{cas}^{-1} = k_\perp} v_k = |\omega_I|$$

Critical balance (GS95)

Ion-neutral collisional damping

$$|\omega_I| \sim k_\parallel^2 V_A^2 / \rho_i$$

Neutral viscous damping

$$|\omega_I| \sim k^2 \nu_n$$

Super-Alfvenic turbulence

(Lazarian 2006)

$$\boldsymbol{\tau_{cas}^{-1} \sim k_\perp^{2/3}}$$

Sub-Alfvenic turbulence

(Lazarian & Vishniac 1999)

$$\boldsymbol{\tau_{cas}^{-1} \sim k_\perp^{2/3} V_A^{-1/3}}$$

Damping scales of Alfvenic turbulence

IN > NV,

Super-A

$$k_{\mathrm{dam,IN,sup}} = \left(\frac{2\nu_{ni}}{\xi_n}\right)^{\frac{3}{2}} L^{\frac{1}{2}} u_L^{-\frac{3}{2}}$$

Sub-A

$$k_{\mathrm{dam,IN,sub}} = \left(\frac{2\nu_{ni}}{\xi_n}\right)^{\frac{3}{2}} L^{\frac{1}{2}} u_L^{-\frac{3}{2}} M_A^{-\frac{1}{2}}$$

~ $V_A^{1/2}$

NV > IN,

Super-A

$$k_{\mathrm{dam,NV,sup}} = \left(\frac{\xi_n}{2}\right)^{-\frac{3}{4}} \nu_n^{-\frac{3}{4}} L^{-\frac{1}{4}} u_L^{\frac{3}{4}}$$

Sub-A

$$k_{\mathrm{dam,NV,sub}} = \left(\frac{\xi_n}{2}\right)^{-\frac{3}{4}} \nu_n^{-\frac{3}{4}} L^{-\frac{1}{4}} u_L^{\frac{3}{4}} M_A^{\frac{1}{4}}$$

~ $V_A^{-1/4}$

Xu et al. 2016, ApJ, 826, 166

Neutral viscous damping & ion-neutral collisional damping

solar chromosphere ●

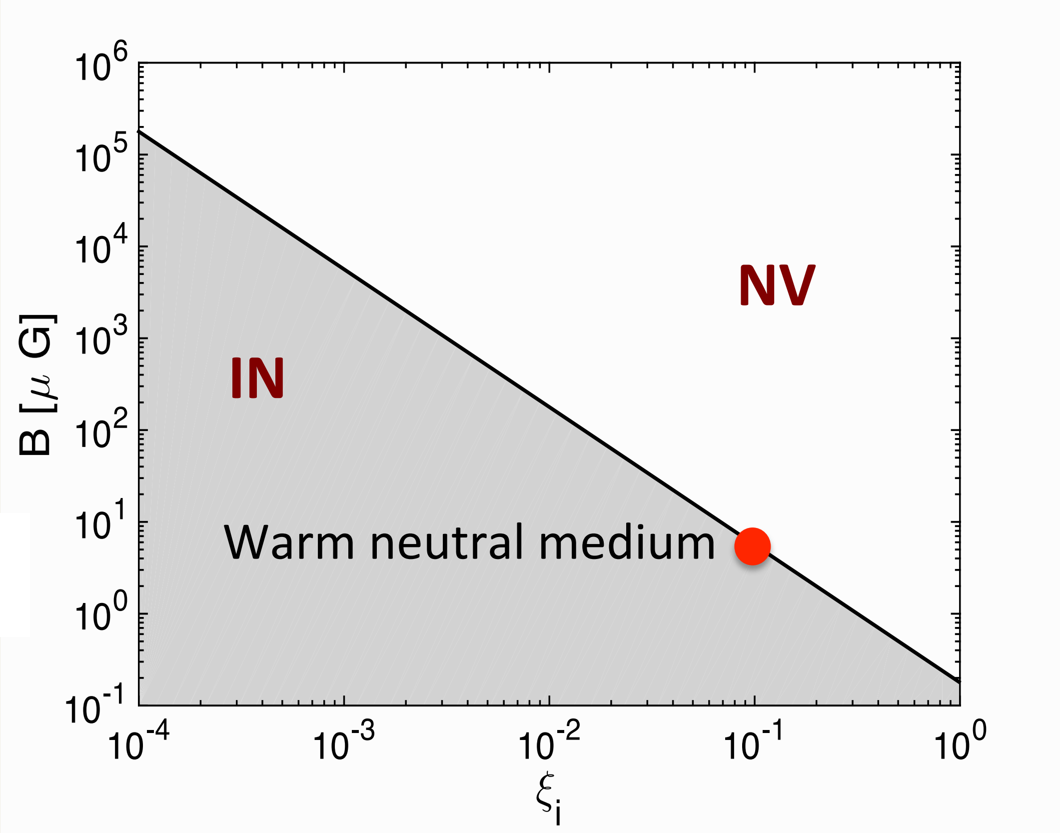
To have **NV > IN,**

Super-Alfvenic turbulence

$$\xi_n \nu_{ni}^2 \nu_n L u_L^{-3} > 0.5$$

Sub-Alfvenic turbulence

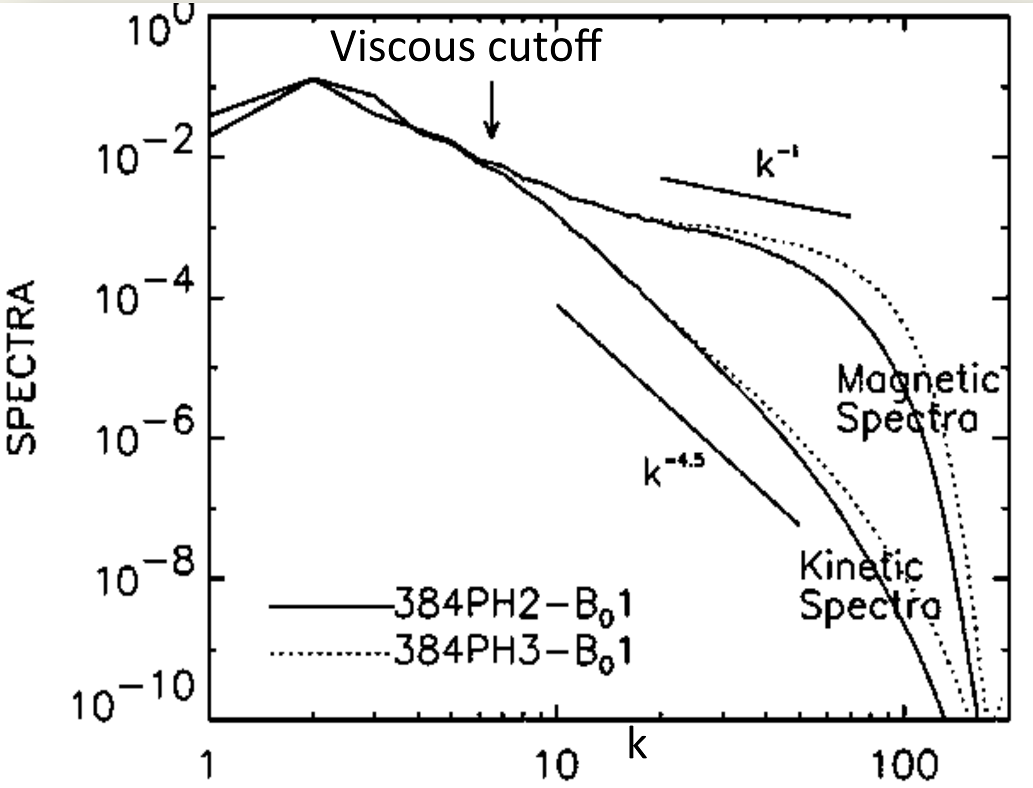
$$\xi_n \nu_{ni}^2 \nu_n L u_L^{-3} M_A^{-1} > 0.5$$

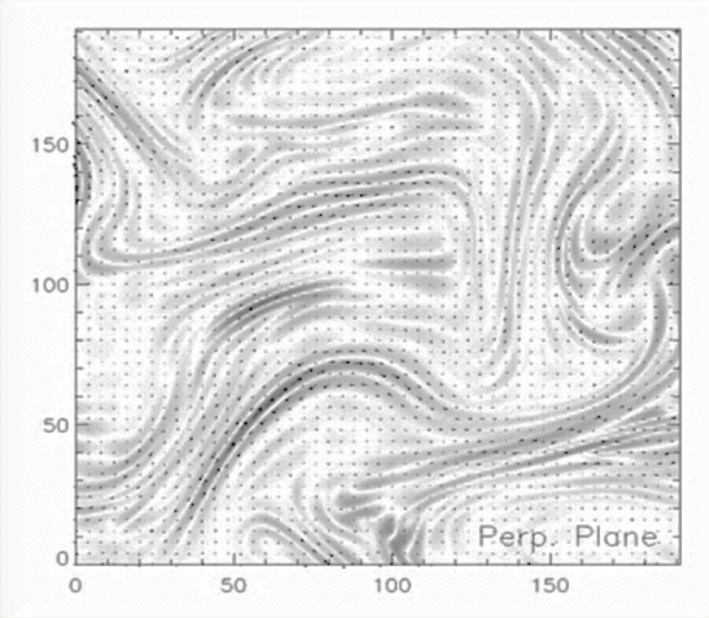


Xu & Lazarian 2017 New J. Phys. 19, 065005

New regime of MHD turbulence

NV dominated regime

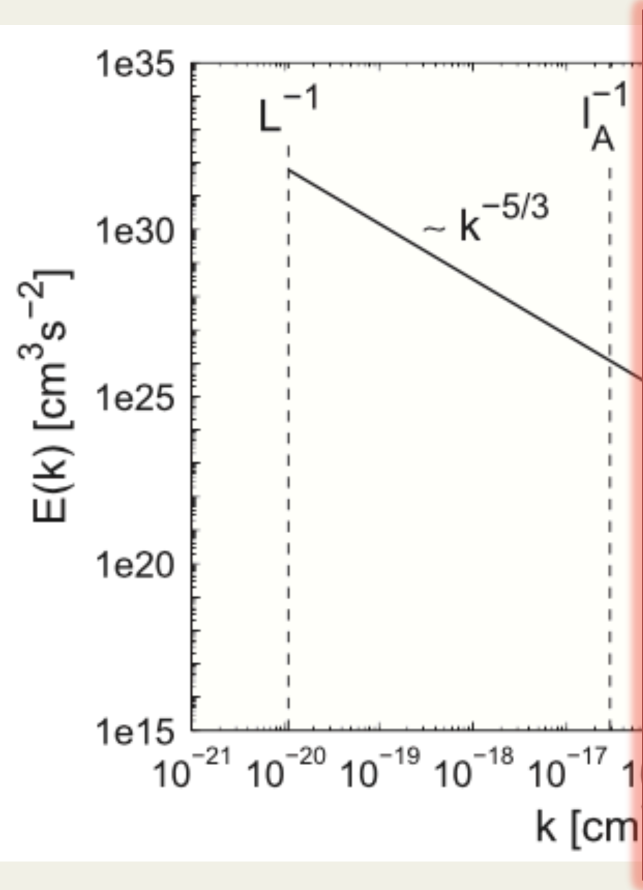


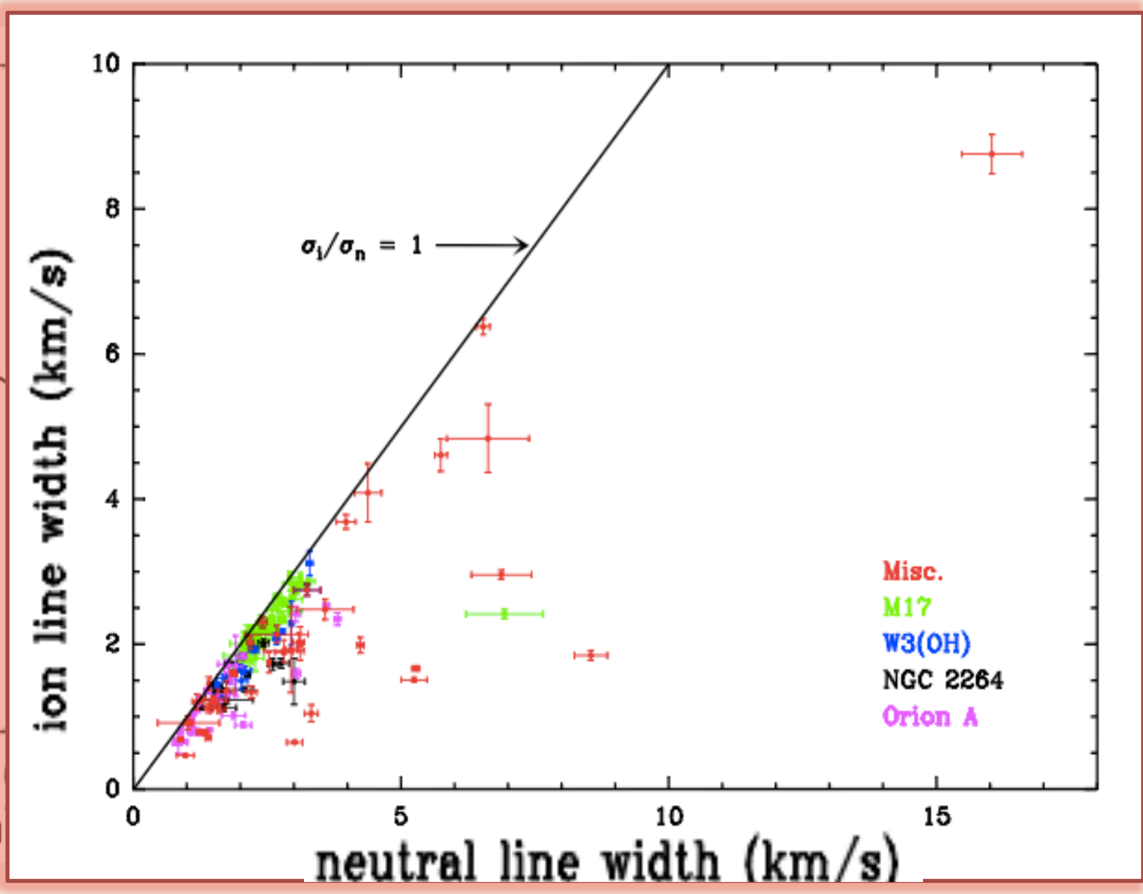


Small ionized and neutral Structures (SINS)

Cho, Lazarian, & Vishniac 2002, 2003

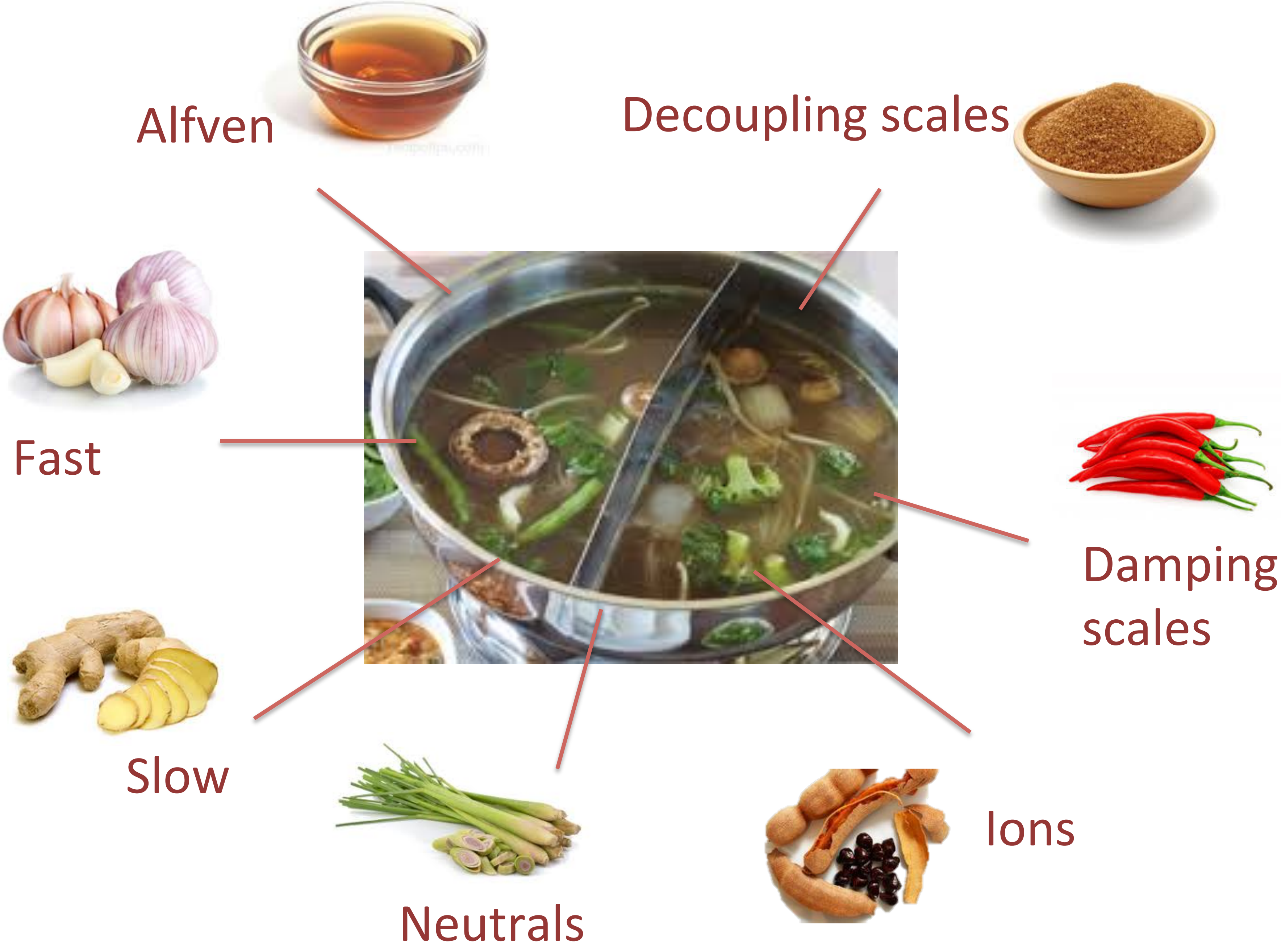
MHD turbulence with IN dominated damping





Houde et al. 2009

Xu et al. 2015, ApJ, 810, 44



Alfven

Decoupling scales

Fast

Damping scales

Slow

Neutrals

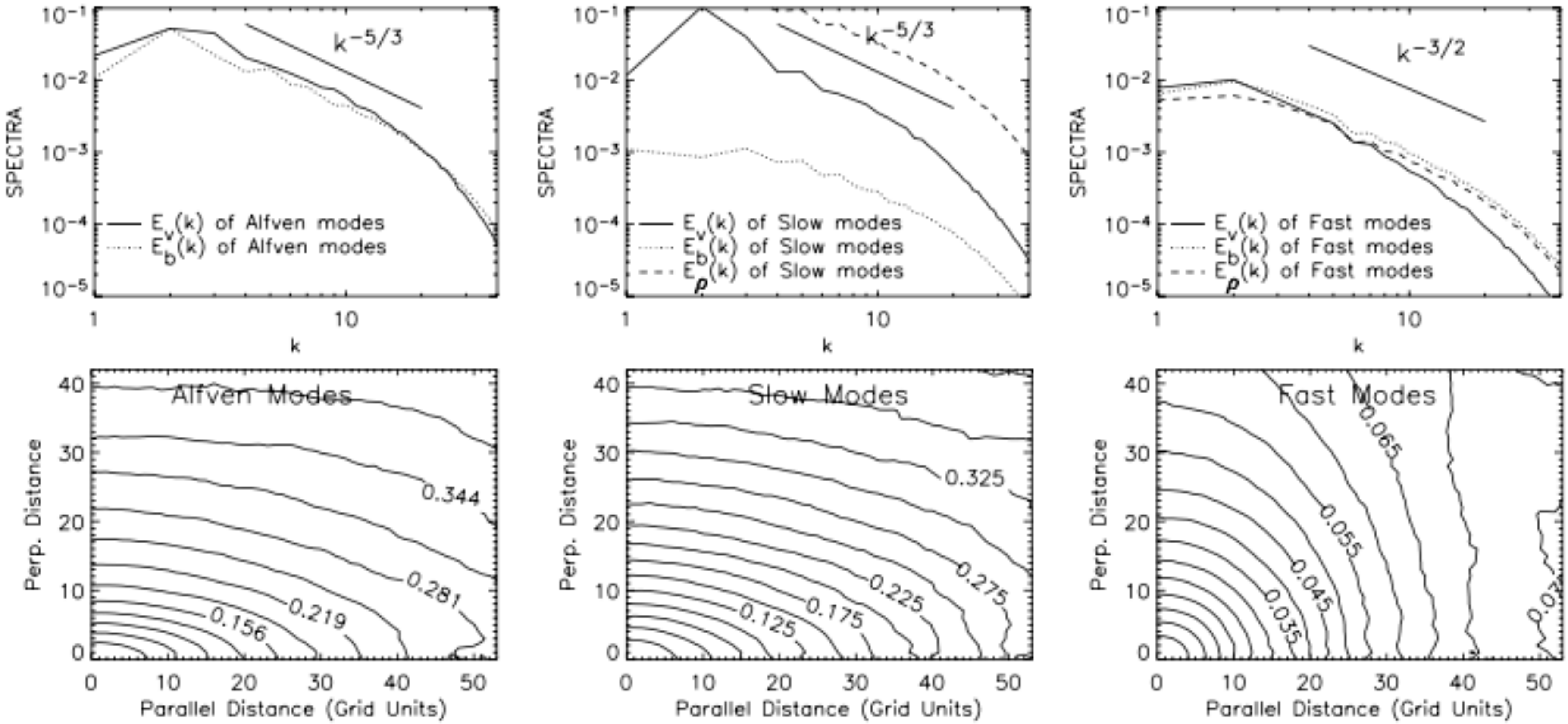
Ions

Mode decomposition of MHD turbulence

Alfven

Slow

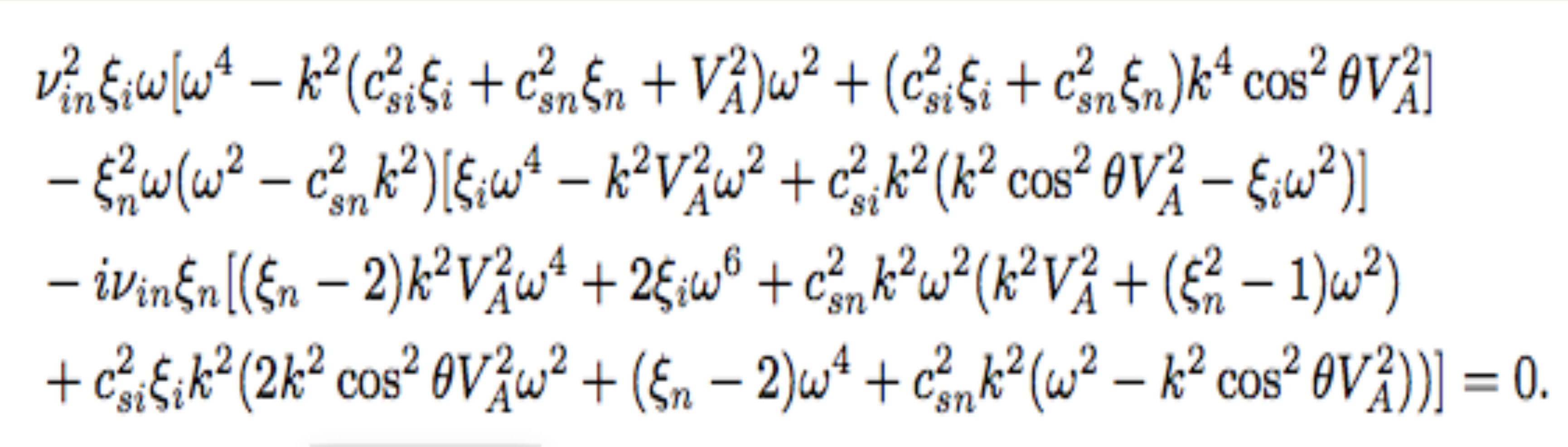
Fast



Cho & Lazarian 2003; Kowal & Lazarian 2010

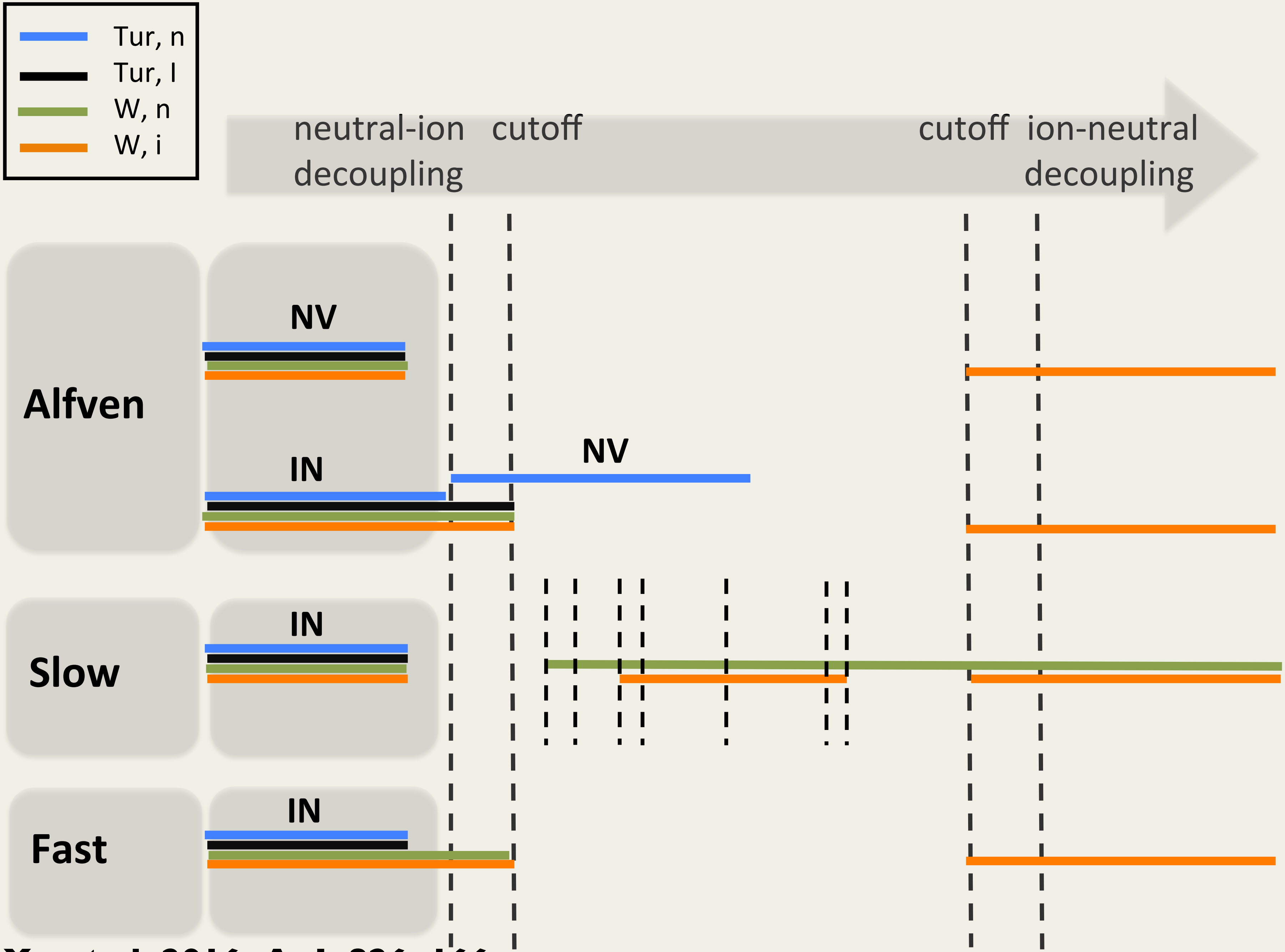
Fast & slow waves in partially ionized gas

Dispersion relations:



$$
\begin{aligned}
&\nu_{in}^2 \xi_i \omega [\omega^4 - k^2(c_{si}^2\xi_i + c_{sn}^2\xi_n + V_A^2)\omega^2 + (c_{si}^2\xi_i + c_{sn}^2\xi_n)k^4\cos^2\theta V_A^2] \\
&- \xi_n^2 \omega(\omega^2 - c_{sn}^2 k^2)[\xi_i\omega^4 - k^2V_A^2\omega^2 + c_{si}^2k^2(k^2\cos^2\theta V_A^2 - \xi_i\omega^2)] \\
&- i\nu_{in}\xi_n[(\xi_n - 2)k^2V_A^2\omega^4 + 2\xi_i\omega^6 + c_{sn}^2k^2\omega^2(k^2V_A^2 + (\xi_n^2 - 1)\omega^2) \\
&+ c_{si}^2\xi_i k^2(2k^2\cos^2\theta V_A^2\omega^2 + (\xi_n - 2)\omega^4 + c_{sn}^2k^2(\omega^2 - k^2\cos^2\theta V_A^2))] = 0.
\end{aligned}
$$

Zaqarashvili et al., 2011



Tur, n

Tur, I

W, n

W, i

neutral-ion decoupling

cutoff

cutoff

ion-neutral decoupling

Alfven

NV

IN

NV

Slow

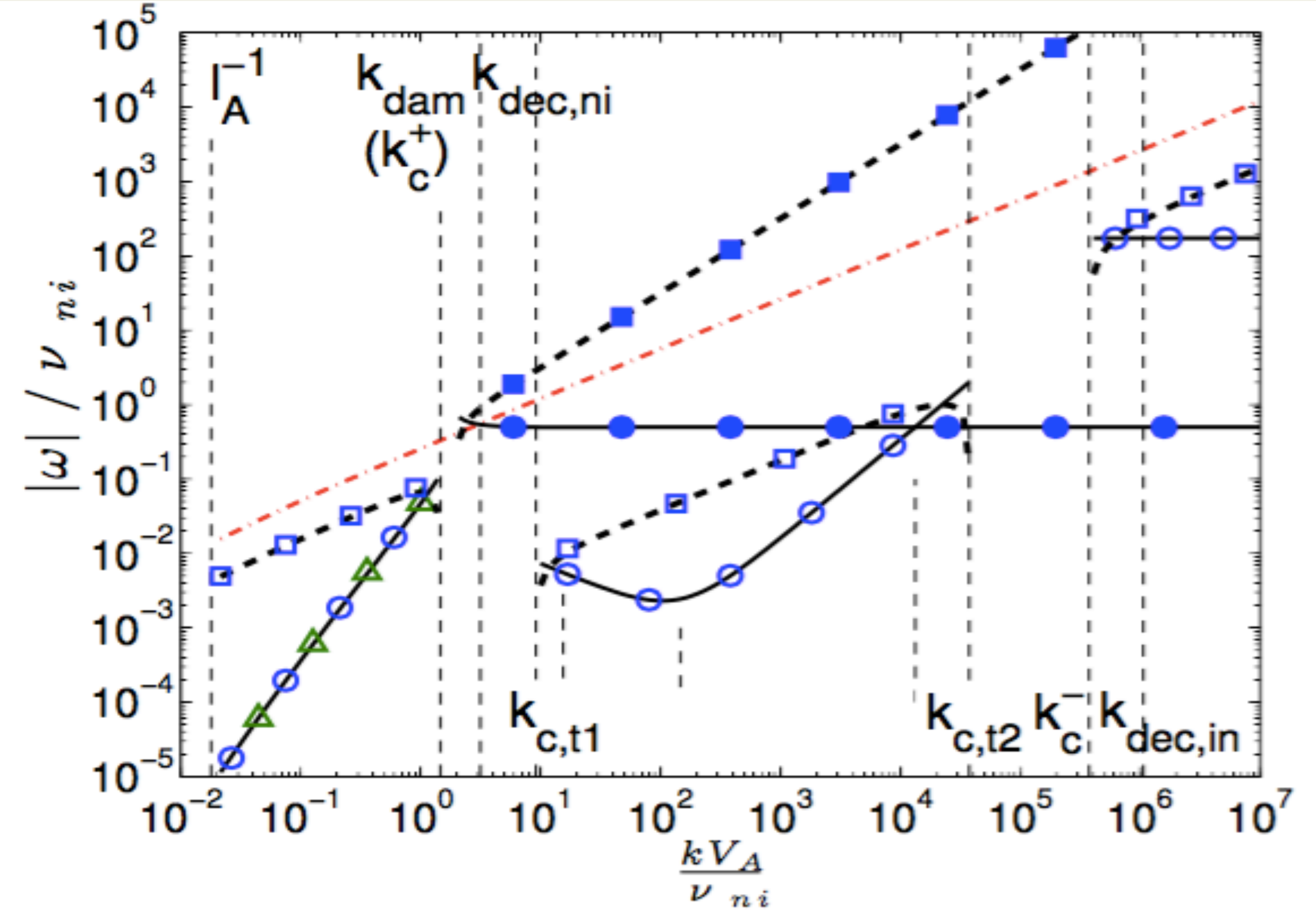
IN

Fast

IN

Xu et al. 2016, ApJ, 826, 166

Slow waves in partially ionized gas



Wave frequency

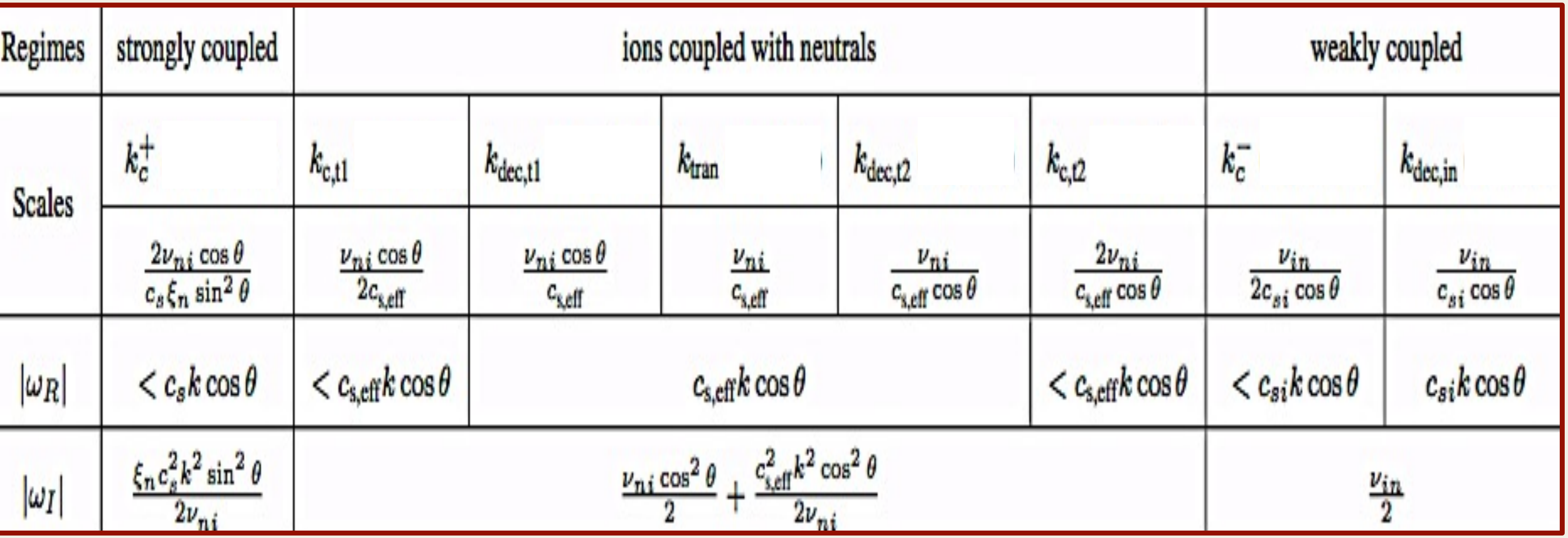
Wave number

Xu et al. 2016, ApJ, 826, 166

Slow waves in partially ionized gas

$$k_{\rm tran} = \frac{k_{\rm dec,t1}}{\cos\theta} = \frac{2k_{\rm c,t1}}{\cos\theta} \approx \frac{\xi_n k_c^+}{2\sqrt{\xi_i}\cos\theta}$$

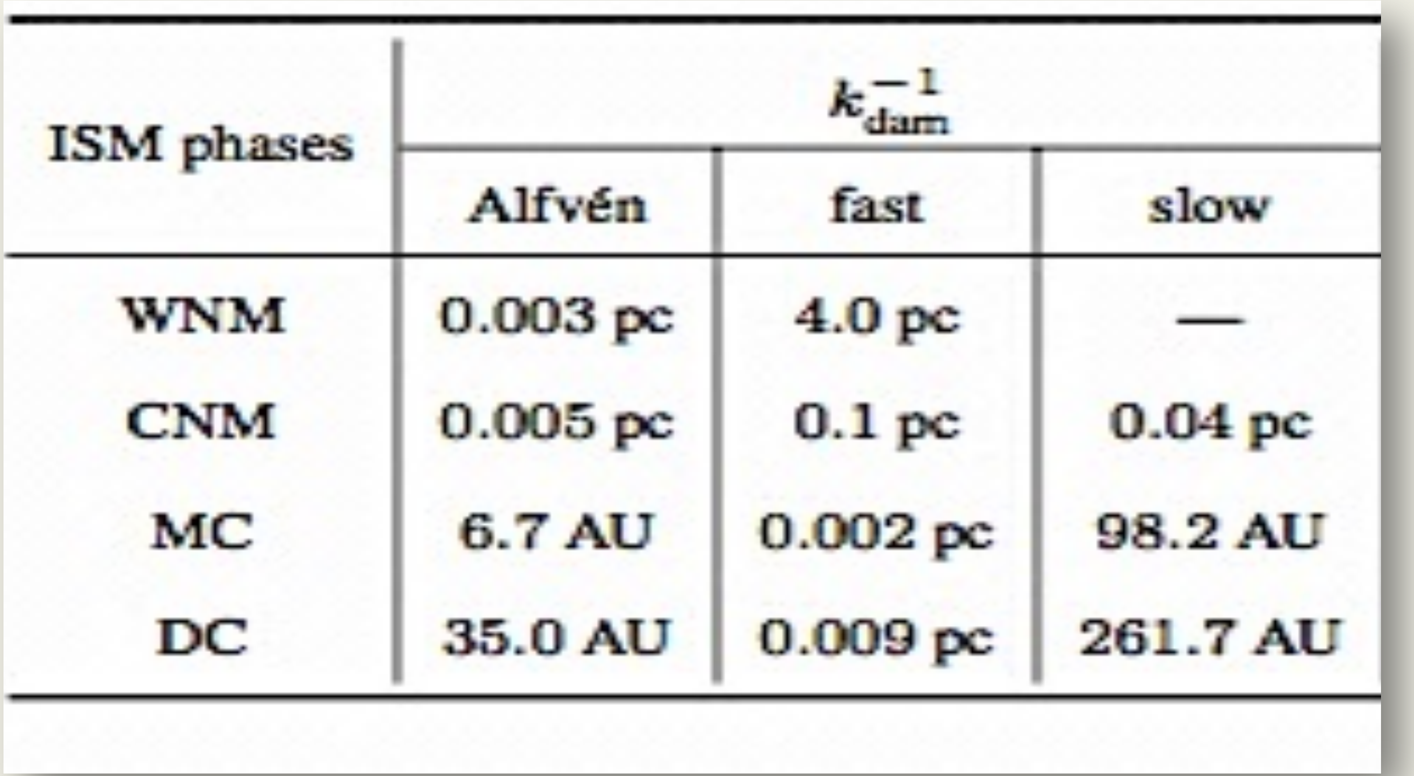
$$= k_{\rm dec,t2}\cos\theta = \frac{k_{\rm c,t2}\cos\theta}{2} = \frac{2\sqrt{\xi_i}\cos\theta k_c^-}{\xi_n}$$



Regimes	strongly coupled	ions coupled with neutrals					weakly coupled	
Scales	k_c^+	$k_{\rm c,t1}$	$k_{\rm dec,t1}$	$k_{\rm tran}$	$k_{\rm dec,t2}$	$k_{\rm c,t2}$	k_c^-	$k_{\rm dec,in}$
	$\frac{2\nu_{ni}\cos\theta}{c_s\xi_n\sin^2\theta}$	$\frac{\nu_{ni}\cos\theta}{2c_{\rm s,eff}}$	$\frac{\nu_{ni}\cos\theta}{c_{\rm s,eff}}$	$\frac{\nu_{ni}}{c_{\rm s,eff}}$	$\frac{\nu_{ni}}{c_{\rm s,eff}\cos\theta}$	$\frac{2\nu_{ni}}{c_{\rm s,eff}\cos\theta}$	$\frac{\nu_{in}}{2c_{si}\cos\theta}$	$\frac{\nu_{in}}{c_{si}\cos\theta}$
$\lvert\omega_R\rvert$	$< c_s k\cos\theta$	$< c_{\rm s,eff}k\cos\theta$	$c_{\rm s,eff}k\cos\theta$			$< c_{\rm s,eff}k\cos\theta$	$< c_{si}k\cos\theta$	$c_{si}k\cos\theta$
$\lvert\omega_I\rvert$	$\frac{\xi_n c_s^2 k^2\sin^2\theta}{2\nu_{ni}}$	$\frac{\nu_{ni}\cos^2\theta}{2} + \frac{c_{\rm s,eff}^2 k^2\cos^2\theta}{2\nu_{ni}}$					$\frac{\nu_{in}}{2}$	

Xu et al. 2016, ApJ, 826, 166

Damping scales of MHD turbulence in different ISM phases



ISM phases	$k_{\rm dam}^{-1}$		
	Alfvén	fast	slow
WNM	0.003 pc	4.0 pc	—
CNM	0.005 pc	0.1 pc	0.04 pc
MC	6.7 AU	0.002 pc	98.2 AU
DC	35.0 AU	0.009 pc	261.7 AU

Xu et al. 2015, ApJ, 810, 44; 2016, ApJ, 826, 166

Turbulent, magnetized, and partially ionized interstellar medium

MHD turbulence in a partially ionized medium

Alfven (GS95) Slow Fast

Applications related to star formation

B measurement

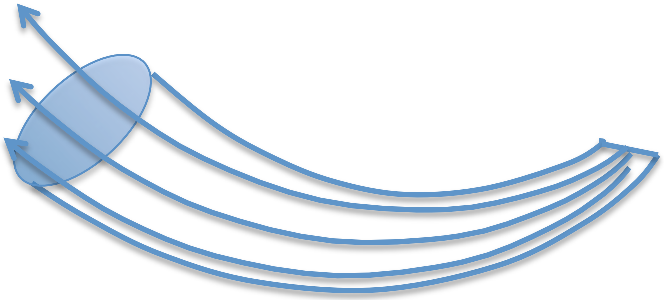
B amplification
(turbulent dynamo)

Xu et al. 2015, ApJ, 810, 44

Xu & Lazarian 2016, ApJ, 833, 215

Application in turbulent dynamo

Dynamo growth of magnetic energy



$B \propto l$

Flux-freezing

Magnetic field

Turbulent eddy

Application in turbulent dynamo

Stretching vs. diffusion

Flux-freezing breakdown:

Below the damping scale

ambipolar diffusion

Above the damping scale

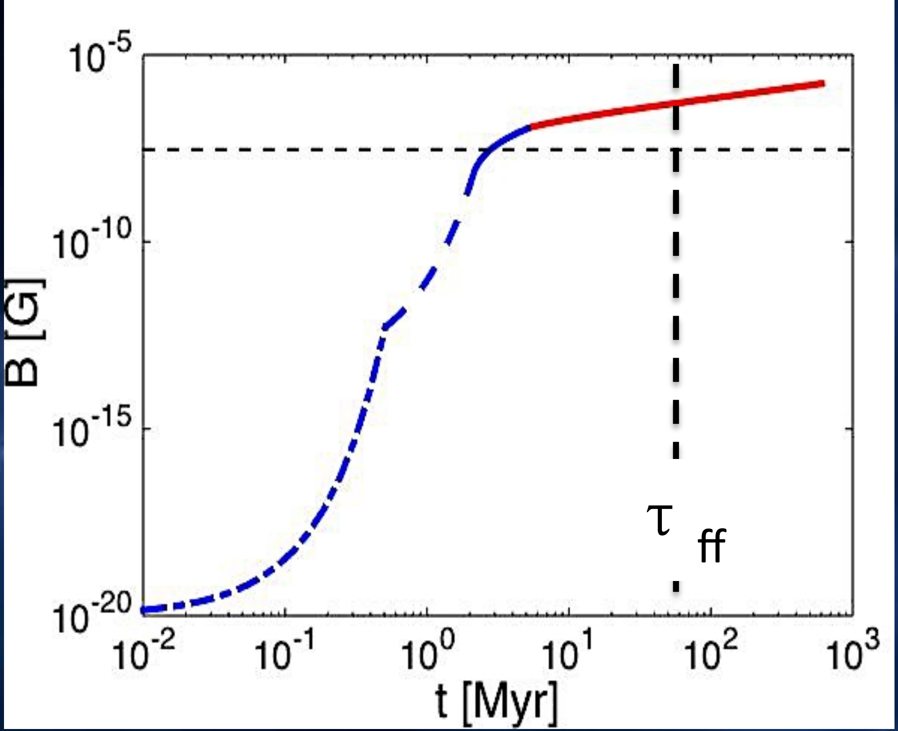
turbulent diffusion

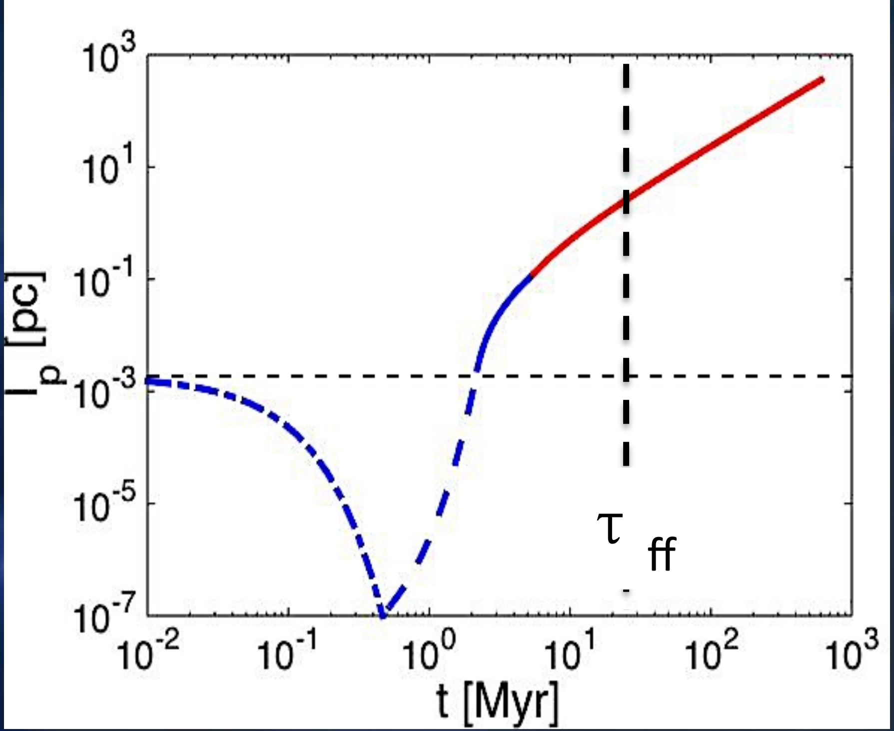
Lazarian & Vishniac 1999; **Xu & Lazarian 2016**

Magnetic fields during the first star formation

Evolution of

magnetic field strength **and** **its characteristic scale**



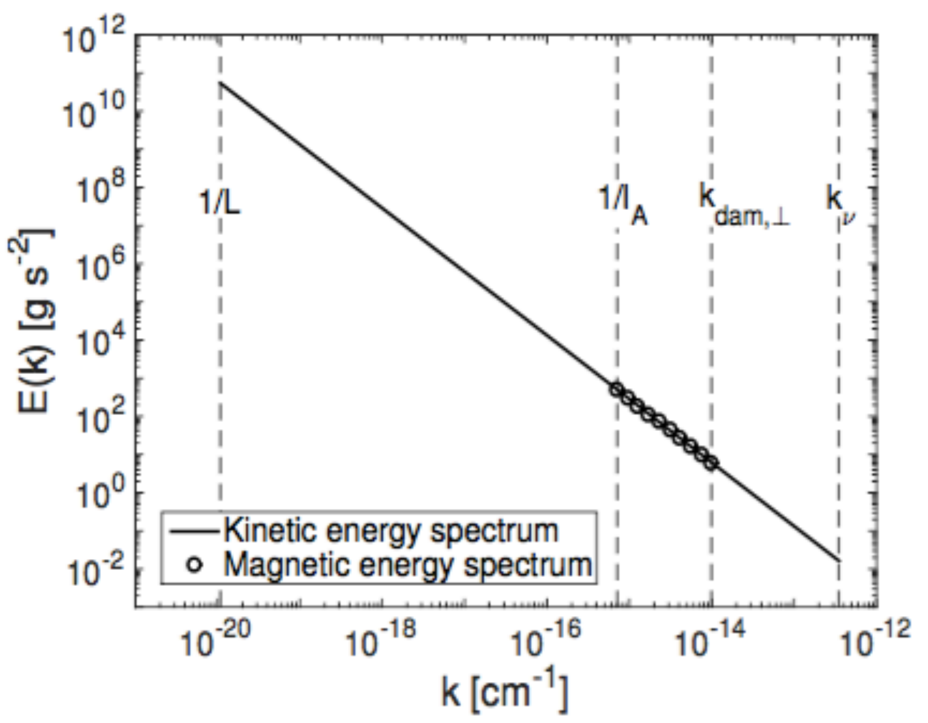


Xu & Lazarian 2016, ApJ, 833, 215

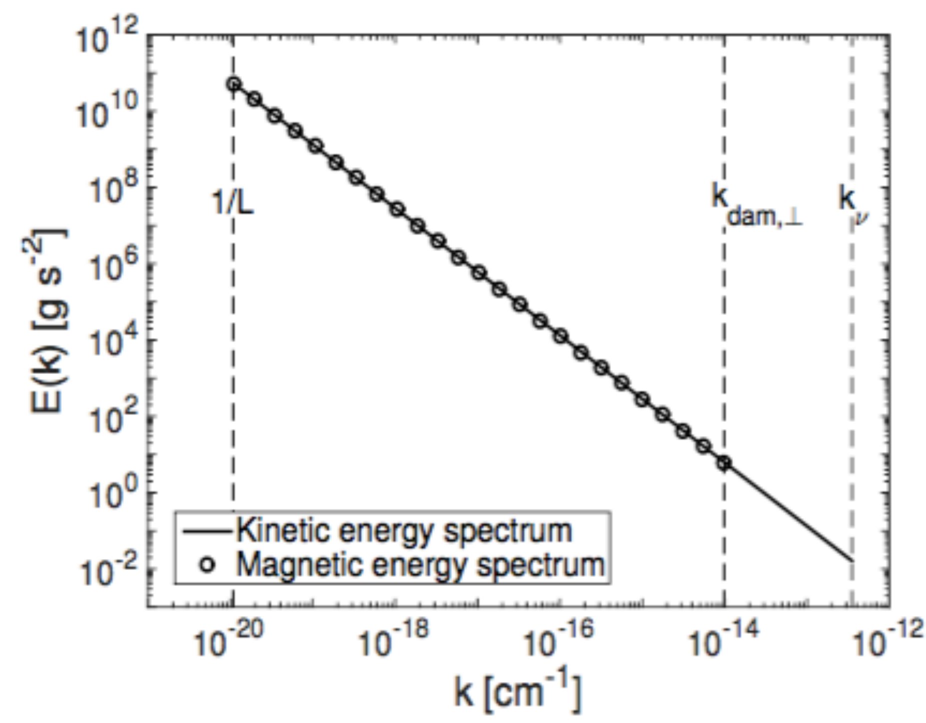
Magnetic fields during the present star formation

Evolution of magnetic energy spectrum

Initial state

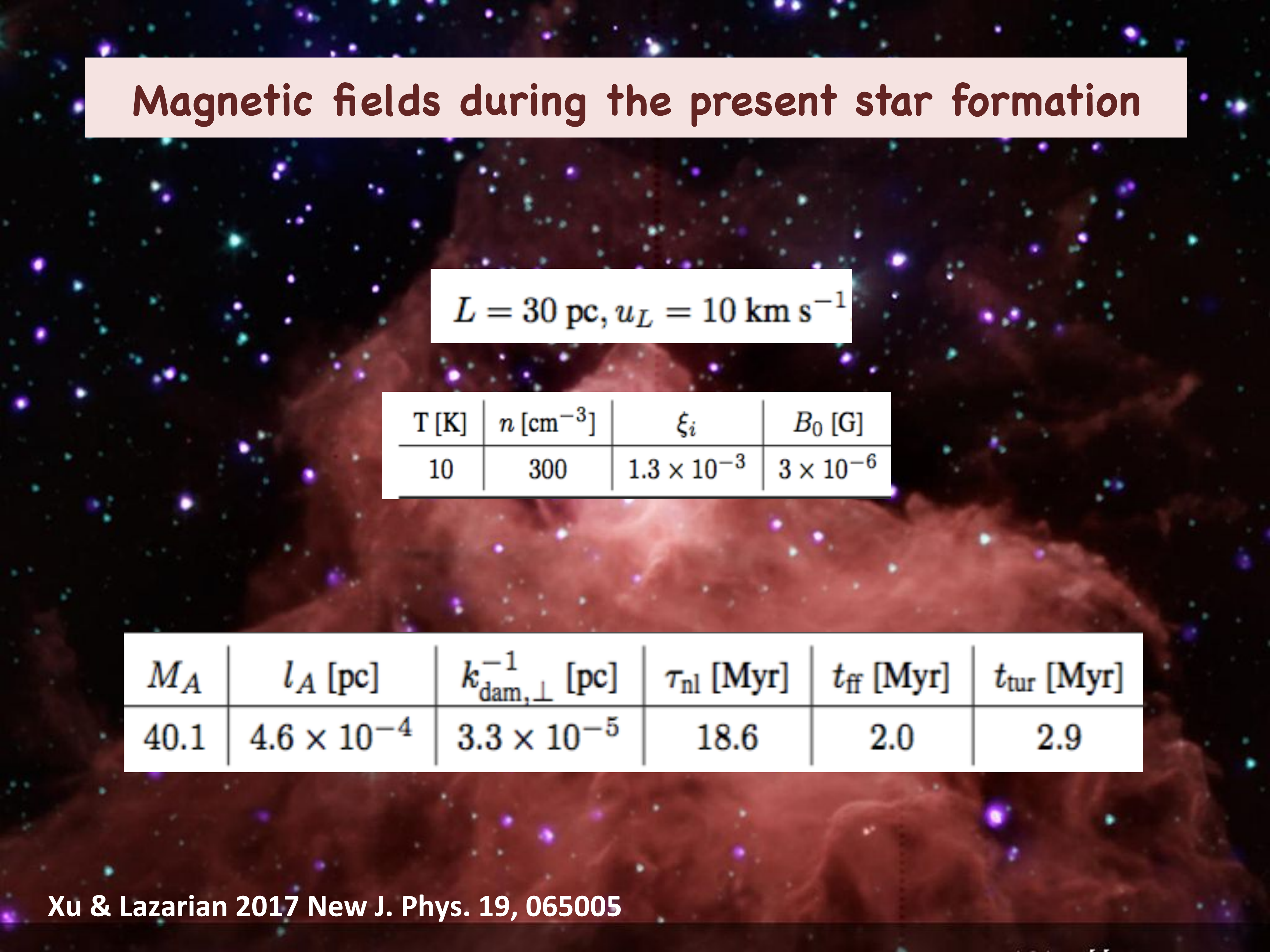


Final state



Xu & Lazarian 2017 New J. Phys. 19, 065005

Magnetic fields during the present star formation



$$L = 30\ \mathrm{pc},\ u_L = 10\ \mathrm{km\ s^{-1}}$$

T [K]	n [cm^{-3}]	ξ_i	B_0 [G]
10	300	1.3×10^{-3}	3×10^{-6}

M_A	l_A [pc]	$k_{\mathrm{dam},\perp}^{-1}$ [pc]	τ_{nl} [Myr]	t_{ff} [Myr]	t_{tur} [Myr]
40.1	4.6×10^{-4}	3.3×10^{-5}	18.6	2.0	2.9

Xu & Lazarian 2017 New J. Phys. 19, 065005

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- Slow
- Fast

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